CSE 390Z: Mathematics for Computation Workshop

Week 2 Workshop Problems

Conceptual Review

(a) Fill in the following definitions.

Tautology:

Contradiction:

Contingency:

(b) What is the contrapositive of $p \to q$? What is the converse of $p \to q$?

Contrapositive:

Converse:

(c) What is a predicate, a domain of discourse, and a quantifier?

(d) When translating to predicate logic, how do you restrict to a smaller domain in a "for all"? How do you restrict to a smaller domain in an "exists"?

1. Review: Propositional Logic Equivalences

Write a chain of logical equivalences to prove the following statements. Note that with propositional logic, you are expected to show all steps, including commutativity and associativity.

(a) $p \rightarrow q \equiv \neg(p \wedge \neg q)$

(b) $\neg p \vee ((q \wedge p) \vee (\neg q \wedge p)) \equiv T$

(c) $((p \wedge q) \rightarrow r) \equiv (p \rightarrow r) \vee (q \rightarrow r)$

2. Review: Boolean Algebra Equivalences

(a) Prove $p' + (p \cdot q) + (q' \cdot p) = 1$ via equivalences.

(b) Prove $(p' + q) \cdot (q + p) = q$ via equivalences.

3. Review: Implications and Vacuous Truth

Alice and Bob's teacher says in class "if a number is prime, then the number is odd." Alice and Bob both believe that the teacher is wrong, but for different reasons.

(a) Alice says "9 is odd and not prime, so the implication is false." Is Alice's justification correct? Why or why not?

(b) Bob says "2 is prime and not odd, so the implication is false." Is Bob's justification correct? Why or why not?

(c) Recall that this is the truth table for implications. Which row does Alice's example correspond to? Which row does Bob's example correspond to?

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

(d) Observe that in order to show that $p \to q$ is false, you need an example where p is true and q is false. Examples where p is false don't disprove the implication! (Nothing to write for this part).

4. Predicate Logic: Warmup

Let the domain of discourse be all animals. Let Cat(x) ::= "x is a cat" and Blue(x) ::= "x is blue". Translate the following statements to English.

(a) $\forall x(\text{Cat}(x) \land \text{Blue}(x))$

(b) $\forall x(\text{Cat}(x) \to \text{Blue}(x))$

(c) $\exists x(\text{Cat}(x) \land \text{Blue}(x))$

(d) Kabir translated the sentence "there exists a blue cat" to $\exists x(\text{Cat}(x) \to \text{Blue}(x))$. This is wrong! Let's understand why.

Use the Law of Implications to rewrite Kabir's translation without the $\to$.

(e) Translate the predicate from (d) back to English. How does this differ from the intended meaning?

(f) This is a warning to be very careful when including an implication nested under an exists! (Nothing to write for this part).

5. Predicate Logic: Domains of Discourse

For the following, find a domain of discourse where the following statement is true and another where it is false. Note that for the arithmetic symbols to make sense, the domains of discourse should be sets of numbers.

(a) $\exists x(2x = 0)$

(b) $\forall x \exists y(x + y = 0)$

(c) $\exists x \forall y(x + y = y)$

6. Predicate Logic: English to Logic

Express the following sentences in predicate logic. The domain of discourse is penguins. You may use the following predicates: Love(x, y) ::= "x loves y", Dances(x) ::= "x dances", Sings(x) ::= "x sings", as well as $=$ and $\neq$.

(a) There is a penguin that every penguin loves.

(b) All penguins that sing love a penguin that does not sing.

(c) There is exactly one penguin that dances.

(d) There exists a penguin that loves itself, but hates (does not love) every other penguin.

7. Predicate Logic: Logic to English

Translate the following sentences to English. Assume the same predicates and domain of discourse as the previous problem.

(a) $\neg\exists x(\text{Dances}(x))$

(b) $\exists x\forall y(\text{Loves}(x, y))$

(c) $\forall x(\text{Dances}(x) \to \exists y(\text{Loves}(y, x)))$

(d) $\exists x\forall y((\text{Dances}(y) \land \text{Sings}(y)) \to \text{Loves}(x, y))$

8. Challenge: Predicate Translation

Translate "You can fool all of the people some of the time, and you can fool some of the people all of the time, but you can't fool all of the people all of the time" into predicate logic. Then, negate your translation. Then, translate the negation back into English.

Hint: Let the domain of discourse be all people and all times, and let $P(x, y)$ be the statement "You can fool person x at time y". You can get away with not defining any other predicates if you use P.