

CSE 390Z: Mathematics for Computation Workshop

QuickCheck: Negating Quantifiers Solutions

Please submit a response to the following questions on Gradescope. We do not grade on accuracy, so please submit your best attempt. You may either typeset your responses or hand-write them. Note that hand-written solutions must be legible to be graded.

We have created **this template** if you choose to typeset with Latex. **This guide** has specific information about scanning and uploading pdf files to Gradescope.

0. Negating Quantifiers

The domain of discourse is movies and actors. The following predicates are defined: $Movie(x) ::= x$ is a movie, $Actor(x) ::= x$ is an actor, $Features(x, y) ::= x$ features y .

Two of your TAs, Karim and Megan, have been tasked with translating the sentence "Not every actor has been featured in a movie" to predicate logic.

This was Karim's translation: $\neg\forall x(Actor(x) \rightarrow \exists y(Movie(y) \wedge Features(y, x)))$

This was Megan's translation: $\exists x(Actor(x) \wedge \forall y(Movie(y) \rightarrow \neg Features(y, x)))$

- (a) Isabel claims that Karim and Megan are both correct. Do you agree with Isabel?

Solution:

Yes, both translations are correct.

- (b) Use a chain of predicate logic equivalences to prove that the two translations are equivalent.

Hint: You may wish to use DeMorgan's Law for Predicates and the Law of Implication.

Solution:

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|--|-------------------------------|
| $\neg\forall x(Actor(x) \rightarrow \exists y(Movie(y) \wedge Features(y, x)))$ | |
| $\equiv \exists x(\neg(Actor(x) \rightarrow \exists y(Movie(y) \wedge Features(y, x))))$ | DeMorgan's Law for Predicates |
| $\equiv \exists x(\neg(\neg Actor(x) \vee \exists y(Movie(y) \wedge Features(y, x))))$ | Law of Implications |
| $\equiv \exists x(\neg\neg Actor(x) \wedge \neg\exists y(Movie(y) \wedge Features(y, x)))$ | DeMorgan's Law |
| $\equiv \exists x(Actor(x) \wedge \neg\exists y(Movie(y) \wedge Features(y, x)))$ | Double Negation |
| $\equiv \exists x(Actor(x) \wedge \forall y(\neg(Movie(y) \wedge Features(y, x))))$ | DeMorgan's Law for Predicates |
| $\equiv \exists x(Actor(x) \wedge \forall y(\neg Movie(y) \vee \neg Features(y, x)))$ | DeMorgan's Law |
| $\equiv \exists x(Actor(x) \wedge \forall y(Movie(y) \rightarrow \neg Features(y, x)))$ | Law of Implications |

1. Video Solution

Watch **this video** on the solution **after** making an initial attempt. Then, answer the following questions.

- (a) What is one thing you took away from the video solution?

(b) What topic from the quick check or lecture would you most like to review in workshop?