CSE 390Z: Mathematics for Computation Workshop

Practice 311 Midterm

Name: ______________________________

UW ID: ______________________________

Instructions:

- This is a **simulated practice midterm**. You will **not** be graded on your performance on this exam.
- Nevertheless, please treat this as if it is a real exam. That means that you may not discuss with your neighbors, reference outside material, or use your devices during the next 60 minute period.
- If you get stuck on a problem, consider moving on and coming back later. In the actual exam, there will likely be opportunity for partial credit.
- There are 5 problems on this exam, totaling 80 points.

1. Predicate Translation [15 points]

Let the domain of discourse be novels, comic books, movies, and TV shows. Translate the following statements to predicate logic, using the following predicates:

Novel(x) := x is a novel
Comic(x) := x is a comic book
Movie(x) := x is a movie
Show(x) := x is a TV show
Adaptation(x, y) := x is an adaptation of y

(a) (5 points) A novel cannot be adapted into both a movie and a TV show.

(b) (5 points) Every movie is an adaptation of a novel or a comic book.

(c) (5 points) Every novel has been adapted into exactly one movie.

2. Canonical Forms [15 points]
The boolean function f takes in three boolean inputs x_1, x_2, x_3, and outputs $\neg((x_1 \oplus x_2) \wedge x_3)$.

Note: You may write your solutions using boolean algebra or propositional logic notation.

(a) (5 points) Draw a truth table for f.

(b) (5 points) Write a propositional logic expression for f in DNF form (ORs of ANDs). Do not try to simplify.

(c) (5 points) Write a propositional logic expression for f in CNF form (ANDs of ORs). Do not try to simplify.

3. Number Theory Proof [20 points]

Recall this definition of odd: $\text{Odd}(x) := \exists y(x = 2y + 1)$. Write an English proof to show that for all odd integers k, the statement $8 \mid k^2 - 1$ holds.

Hint: At some point in your proof, you'll need to show that for any integer a, $a(a + 1)$ is even. When you reach this point, feel free to break your proof up into the case where a is even, and the case where a is odd.

4. Sets [10 points]
Determine if the following claims are true or false. Then explain your reasoning in 1-3 sentences.
You may include images or examples in your explanation. **You do not need to give a formal proof or disproof.**

(a) (5 points) For all sets A, B: $(A \cup B) \setminus (A \cap B) = (A \setminus B) \cup (B \setminus A)$.

(b) (5 points) For all sets A, B: $\mathcal{P}(A) \times \mathcal{P}(B) \subseteq \mathcal{P}(A \times B)$.

5. Induction [20 points]
Prove by induction that $(1+\pi)^n > 1 + n\pi$ for all integers $n \geq 2$.