# CSE 390Z: Mathematics for Computation Workshop

# **Practice 311 Midterm**

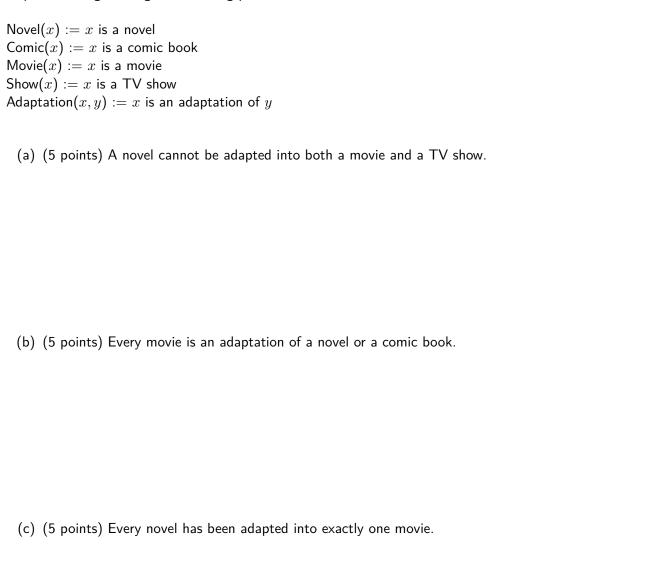
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#### Instructions:

- This is a **simulated practice midterm**. You will **not** be graded on your performance on this exam.
- Nevertheless, please treat this as if it is a real exam. That means that you may not discuss with your neighbors, reference outside material, or use your devices during the next 60 minute period.
- If you get stuck on a problem, consider moving on and coming back later. In the actual exam, there will likely be opportunity for partial credit.
- There are 5 problems on this exam, totaling 80 points.

#### 1. Predicate Translation [15 points]

Let the domain of discourse be novels, comic books, movies, and TV shows. Translate the following statements to predicate logic, using the following predicates:



#### 2. Canonical Forms [15 points]

The boolean function f takes in three boolean inputs  $x_1, x_2, x_3$ , and outputs  $\neg((x_1 \oplus x_2) \land x_3)$ .

Note: You may write your solutions using boolean algebra or propositional logic notation.

(a) (5 points) Draw a truth table for f.

(b) (5 points) Write a propositional logic expression for f in DNF form (ORs of ANDs). Do not try to simplify.

(c) (5 points) Write a propositional logic expression for f in CNF form (ANDs of ORs). Do not try to simplify.

## 3. Number Theory Proof [20 points]

Recall this definition of odd:  $Odd(x) := \exists y(x=2y+1)$ . Write an English proof to show that for all odd integers k, the statement  $8 \mid k^2-1$  holds.

**Hint:** At some point in your proof, you'll need to show that for any integer a, a(a+1) is even. When you reach this point, feel free to break your proof up into the case where a is even, and the case where a is odd.

## **4. Sets** [10 points]

Determine if the following claims are true or false. Then explain your reasoning in 1-3 sentences.

You may include images or examples in your explanation. You do not need to give a formal proof or disproof.

(a) (5 points) For all sets A, B:  $(A \cup B) \setminus (A \cap B) = (A \setminus B) \cup (B \setminus A)$ .

(b) (5 points) For all sets  $A, B: \mathcal{P}(A) \times \mathcal{P}(B) \subseteq \mathcal{P}(A \times B)$ .

**5. Induction** [20 points] Prove by induction that  $(1+\pi)^n>1+n\pi$  for all integers  $n\geq 2$ .