# CSE 390Z: Mathematics for Computation Workshop

## Practice 311 Final

Name: ________________________________________

UW ID: ________________________________________

**Instructions:**

- This is a **simulated practice final**. You will **not** be graded on your performance on this exam.
- Nevertheless, please treat this as if it is a real exam. That means that you may not discuss with your neighbors, reference outside material, or use your devices during the next 60 minute period.
- If you get stuck on a problem, consider moving on and coming back later. In the actual exam, there will likely be opportunity for partial credit.
- There are 5 problems on this exam, totaling 100 points.

**1. True or False** [30 points]

For the following questions, determine whether the statement is true or false. Then provide 1-3 sentences of explanation. Your explanations **do not** need to be full or formal proofs.

(a) (5 points) "$p$ only if $q$" and "$q$ is necessary for $p$", are both best translated as $p \to q$.

(b) (5 points) One way to prove that $p \to q$ is true is to show that the converse, $q \to p$, is false.

(c) (5 points) The implication $\forall y \exists x$ P$(x, y) \to \exists x \forall y$ P$(x, y)$ is true regardless of what the predicate P is.

(d) (5 points) Suppose $a, b, m, n$ are all integers greater than 1. If $a \equiv_m b$ and $m \mid n$, then $a \equiv_n b$.

(e) (5 points) Suppose $A, B$ are sets. If $\mathcal{P}(A) \subseteq \mathcal{P}(B)$, then $A \subseteq B$.

(f) (5 points) Strong induction proofs always require more than one base case.

**2. All the Machines!** [15 points]

Let the alphabet be $\Sigma = \{a, b\}$. Consider the language $L = \{w \in \Sigma^* : \text{every } a \text{ has a } b \text{ two characters later}\}$. In other words, $L$ is the language of all strings in the alphabet $a, b$ where after any $a$, the character after the $a$ can be anything, but the character after that one must be a $b$.

Some strings in $L$ include $\varepsilon$, $abb$, $aabb$, $bbbbabb$. Some strings not in $L$ include $a$, $ab$, $aab$, $ababb$. Notice that the last two characters of the string cannot be an $a$.

(g) (5 points) Give a regular expression that represents $L$.

(h) (5 points) Give a CFG that represents $L$.

(i) (5 points) Give a DFA that represents $L$.

## 3. Induction I (20 points)

Let the function $f : \mathbb{N} \to \mathbb{N}$ be defined as follows:

$f(0) = 2$
$f(1) = 7$
$f(n) = f(n-1) + 2f(n-2)$ for $n \geq 2$
Prove that $f(n) = 3 * 2^n + (-1)^{n+1}$ for all integers $n \geq 0$ using strong induction.

## 4. Induction II (20 points)

Let the set $S$ be recursively defined as follows:
**Basis:** $(0, 0) \in S$
**Recursive Step:** If $(x, y) \in S$, then $(x + 2, y + 4) \in S$ and $(x + 4, y + 8) \in S$.

Prove that for all $(x, y) \in S$, $x + y$ is divisible by 3.

**5. Irregularity (15 points)**

Prove that the language $L = \{10^x 10^{x+1} 1 : x \geq 0\}$ is not regular.