CSE 390Z: Mathematics for Computation Workshop

Week 1 Workshop Problems Solutions

Conceptual Review

Space to take notes from the Conceptual Review.

Solution:

1. Translation: Running from my problems

Define a set of three atomic propositions, and use them to translate the following sentences.

(i) I am going for a run and it is snowing, or it is not snowing.

(ii) If it's snowing and it's Friday, I am not going for a run.

(iii) I am going for a run only if it is not Friday.

Solution:

p: I am going for a run
q: It is snowing
r: It is Friday

(i) $(p \wedge q) \vee \neg q$

(ii) $(q \wedge r) \rightarrow \neg p$

(iii) $p \rightarrow \neg r$

2. Translation: Age is just a number

Define a set of two atomic propositions, and use them to translate the following sentences.

(i) If Kai is older than thirty, then Kai is older than twenty.

(ii) Kai is older than thirty only if Kai is older than twenty.

(iii) Whenever Kai is older than thirty, Kai is older than twenty.

(iv) Kai being older than twenty is necessary for Kai to be older than thirty.

Solution:

p: Kai is older than thirty
q: Kai is older than twenty

(i) $p \rightarrow q$

(ii) $p \rightarrow q$

(iii) $p \rightarrow q$

(iv) $p \rightarrow q$

3. Truth Table

Draw a truth table for $(p \to \neg q) \to (r \oplus q)$

Solution:

p	q	r	$\neg q$	$p \to \neg q$	$r \oplus q$	$(p \to \neg q) \to (r \oplus q)$
T	T	T	F	F	F	T
T	T	F	F	F	T	T
T	F	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	T	F	F	T	T	T
F	F	T	T	T	T	T
F	F	F	T	T	F	F

4. Proving Equivalences

We wish to prove that the following is true for all values of the propositions p and q (i.e. that it is a tautology):

$$\neg p \vee ((q \wedge p) \vee (\neg q \wedge p))$$

The following chain of equivalences does this, but it is missing citations for which rules are used. Fill in the blanks with names of the logic equivalences used at each step.

Hint: Reference the Logical Equivalences sheet under the Resources tab on the CSE 311 Course Website.

$$\begin{aligned}
\neg p \vee ((q \wedge p) \vee (\neg q \wedge p)) &\equiv \neg p \vee ((p \wedge q) \vee (\neg q \wedge p)) && \underline{\hspace{6cm}} \\
&\equiv \neg p \vee ((p \wedge q) \vee (p \wedge \neg q)) && \underline{\hspace{6cm}} \\
&\equiv \neg p \vee (p \wedge (q \vee \neg q)) && \underline{\hspace{6cm}} \\
&\equiv \neg p \vee (p \wedge T) && \underline{\hspace{6cm}} \\
&\equiv \neg p \vee p && \underline{\hspace{6cm}} \\
&\equiv p \vee \neg p && \underline{\hspace{6cm}} \\
&\equiv T && \underline{\hspace{6cm}}
\end{aligned}$$

Solution:

$$\begin{aligned}
\neg p \vee ((q \wedge p) \vee (\neg q \wedge p)) &\equiv \neg p \vee ((p \wedge q) \vee (\neg q \wedge p)) && \text{Commutativity} \\
&\equiv \neg p \vee ((p \wedge q) \vee (p \wedge \neg q)) && \text{Commutativity} \\
&\equiv \neg p \vee (p \wedge (q \vee \neg q)) && \text{Distributivity} \\
&\equiv \neg p \vee (p \wedge T) && \text{Negation} \\
&\equiv \neg p \vee p && \text{Identity} \\
&\equiv p \vee \neg p && \text{Commutativity} \\
&\equiv T && \text{Negation}
\end{aligned}$$

5. To Be or Not to Be Equivalent

For each of the following pairs of propositions, determine if the two propositions are equivalent. If they are, prove it using a chain of logical equivalences. If they are not, find an assignment of p and q on which their truth value differ.

(a) $p \wedge q$ vs. $p \vee q$

Solution:

These are not equivalent. For instance, when $p := T, q := F$, the proposition $p \wedge q$ evaluates to F but $p \vee q$ evaluates to T.

(b) $p \rightarrow q$ vs. $\neg q \rightarrow \neg p$
Note: You may not use the Law of Contrapositive in your justification!

Solution:

These are equivalent. Below is the chain of equivalences.

$$\begin{aligned} p \rightarrow q &\equiv \neg p \vee q && \text{Law of Implication} \\ &\equiv \neg p \vee \neg\neg q && \text{Double Negation} \\ &\equiv \neg\neg q \vee \neg p && \text{Commutativity} \\ &\equiv \neg q \rightarrow \neg p && \text{Law of Implication} \end{aligned}$$

(c) $p \rightarrow q$ vs. $q \rightarrow p$

Solution:

These are not equivalent. For instance, when $p := T, q := F$, the proposition $p \rightarrow q$ evaluates to F but $q \rightarrow p$ evaluates to T.

(d) $p \rightarrow q$ vs. $\neg(p \wedge \neg q)$

Solution:

These are equivalent. Below is the chain of equivalences.

$$\begin{aligned} p \rightarrow q &\equiv \neg p \vee q && \text{Law of Implication} \\ &\equiv \neg p \vee \neg\neg q && \text{Double Negation} \\ &\equiv \neg(p \wedge \neg q) && \text{DeMorgan's Law} \end{aligned}$$

6. Implications and Vacuous Truth

Alice and Bob's teacher says in class "if a number is prime, then the number is odd." Alice and Bob both believe that the teacher is wrong, but for different reasons.

(a) Alice says "9 is odd and not prime, so the implication is false." Is Alice's justification correct? Why or why not?

Solution:

No. She gave an example where the premise (number is prime) is false, and the conclusion is true. This doesn't disprove the claim!

(b) Bob says "2 is prime and not odd, so the implication is false." Is Bob's justification correct? Why or why not?

Solution:

Yes. He gave an example where the premise (number is prime) is true, and the conclusion is false. This does disprove the claim!

(c) Recall that this is the truth table for implications. Which row does Alice's example correspond to? Which row does Bob's example correspond to?

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Solution:

Alice's example corresponds to the row where p is false and q is true. Bob's example corresponds to the row where p is true and q is false.

(d) Observe that in order to show that $p \to q$ is false, you need an example where p is true and q is false. Examples where p is false don't disprove the implication! (Nothing to write for this part).

7. Conditional Logic

You've already seen how implications translate to English and back, but we can also translate them to code. In English, $p \to q$ translates to "If p, then q". We might also translate this (in Java) to a conditional:

```
if (p) {
    q
}
```

Lets say we have propositions p, q, and r. It may be helpful in the following problem to think of p and q as booleans, and r as a print statement.

(a) Convert each of the following propositions to Java (or pseudo-) code:

$$p \to (q \to r)$$

Solution:

```
if (p) {
    if (q) {
        r
    }
}
```

$$(p \land q) \to r$$

Solution:

```
if (p && q) {
    r
}
```

(b) Based on this, do you think $p \to (q \to r)$ and $(p \wedge q) \to r$ are logically equivalent? Why or why not?

Solution:

Yes. We can in fact prove that these are equivalent with a chain of logical equivalences or a truth table. (A great exercise to do to get more practice).

8. Propositions in the wild

Give a real-life example of statements p, q, and r such that p and q together imply r, but neither p nor q alone imply r.

Solution:

There is no single correct answer for this, but here is an example:
p : I'm older than 12
q : I'm younger than 20
r : I'm a teen