# CSE 390Z: Mathematics for Computation Workshop

## **Practice 311 Final**

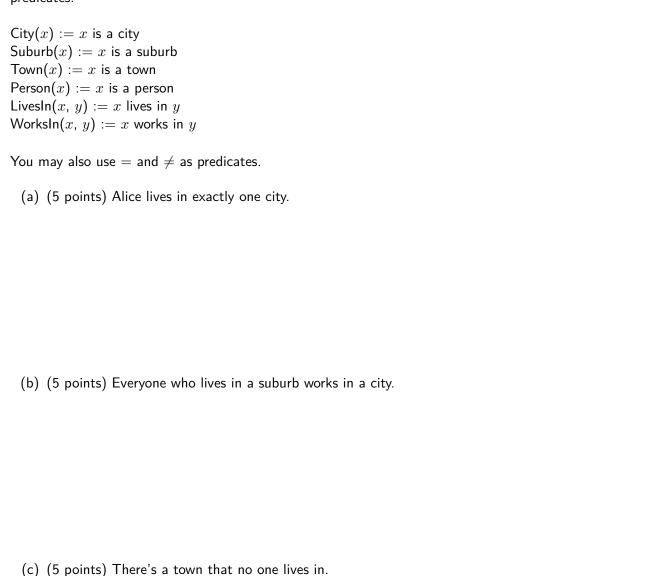
Name:			
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#### Instructions:

- This is a **simulated practice final**. You will **not** be graded on your performance on this exam.
- This final was written to take 50 minutes. The real final will be 110 minutes.
- Nevertheless, please treat this as if it is a real exam. That means that you may not discuss with your neighbors, reference outside material, or use your devices during the next one hour period.
- If you get stuck on a problem, consider moving on and coming back later. In the actual exam, there will likely be opportunity for partial credit.
- There are 5 problems on this exam.

## 1. Predicate Translation [20 points]

Let the domain of discourse be people. Translate the following statements to predicate logic, using the following predicates:



(d) (5 points) Every city is also a town, but not every town is a city.

### 2. All the Machines! [15 points]

Let the alphabet be  $\Sigma = \{a,b\}$ . Consider the language  $L = \{w \in \Sigma^* : \text{every } a \text{ has a } b \text{ two characters later}\}$ . In other words, L is the language of all strings in the alphabet a,b where after any a, the character after the a can be anything, but the character after that one must be a b.

Some strings in L include  $\varepsilon$ , abb, aabb, bbbbabb. Some strings not in L include a, ab, aab, ababb. Notice that the last two characters of the string cannot be an a.

(a) (5 points) Give a regular expression that represents L.

(b) (5 points) Give a CFG that represents L.

(c) (5 points) Give a DFA that represents L.

## **3. Induction** [20 points]

Consider the following recursive definition of  $a_n$ :

$$a_1 = 1$$
 
$$a_2 = 1$$
 
$$a_n = \frac{1}{2}(a_{n-1} + \frac{2}{a_{n-2}})$$
 for  $n > 2$ 

Prove that  $1 \le a_n \le 2$  for all integers  $n \ge 1$ .

## 4. Modular Arithmetic [10 points]

(a) Prove or disprove: If  $a \equiv b \pmod{10}$ , then  $a \equiv b \pmod{5}$ . [5 points]

(b) Prove or disprove: If  $a \equiv b \pmod{10}$ , then  $a \equiv b \pmod{20}$ . [5 points]

5. That's Illegal [20 points] Prove that the set of strings  $\{0^n10^n:n\geq 0\}$  is not regular.