CSE 390Z: Mathematics for Computation Workshop

Practice 311 Final

Name: _____

UW ID: _____

Instructions:

- This is a **simulated practice final**. You will **not** be graded on your performance on this exam.
- This final was written to take 50 minutes. The real final will be 110 minutes.
- Nevertheless, please treat this as if it is a real exam. That means that you may not discuss with your neighbors, reference outside material, or use your devices during the next one hour period.
- If you get stuck on a problem, consider moving on and coming back later. In the actual exam, there will likely be opportunity for partial credit.
- There are 5 problems on this exam.

1. Predicate Translation [20 points]
Let the domain of discourse be people. Translate the following statements to predicate logic, using the following predicates:

City(x) := x is a city
Suburb(x) := x is a suburb
Town(x) := x is a town
Person(x) := x is a person
LivesIn(x, y) := x lives in y
WorksIn(x, y) := x works in y

You may also use $=$ and $\neq$ as predicates.

(a) (5 points) Alice lives in exactly one city.

(b) (5 points) Everyone who lives in a suburb works in a city.

(c) (5 points) There's a town that no one lives in.

(d) (5 points) Every city is also a town, but not every town is a city.

2. All the Machines! [15 points]
Let the alphabet be $\Sigma = \{a, b\}$. Consider the language $L = \{w \in \Sigma^* : \text{every } a \text{ has a } b \text{ two characters later}\}$. In other words, L is the language of all strings in the alphabet a, b where after any a, the character after the a can be anything, but the character after that one must be a b.

Some strings in L include ε, abb, $aabb$, $bbbbabb$. Some strings not in L include a, ab, aab, $ababb$. Notice that the last two characters of the string cannot be an a.

(a) (5 points) Give a regular expression that represents L.

(b) (5 points) Give a CFG that represents L.

(c) (5 points) Give a DFA that represents L.

3. Induction [20 points]
Consider the following recursive definition of a_n:

$$a_1 = 1$$
$$a_2 = 1$$
$$a_n = \frac{1}{2}(a_{n-1} + \frac{2}{a_{n-2}}) \qquad \text{for } n > 2$$

Prove that $1 \leq a_n \leq 2$ for all integers $n \geq 1$.

4. Modular Arithmetic [10 points]

(a) Prove or disprove: If $a \equiv b \pmod{10}$, then $a \equiv b \pmod{5}$. [5 points]

(b) Prove or disprove: If $a \equiv b \pmod{10}$, then $a \equiv b \pmod{20}$. [5 points]

5. That's Illegal [20 points]
Prove that the set of strings $\{0^n 10^n : n \geq 0\}$ is not regular.