CSE 390Z: Mathematics of Computing

Week 9 Workshop

Conceptual Review

Relations definitions: Let R be a relation on A. In other words, $R \subseteq A \times A$. Then:

- R is reflexive iff for all $a \in A$, $(a, a) \in R$.
- R is symmetric iff for all a, b, if $(a, b) \in R$, then $(b, a) \in R$.
- R is antisymmetric iff for all a, b, if $(a, b) \in R$ and $a \neq b$, then $(b, a) \notin R$.
- R is transitive iff for all a, b, if $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$.

1. Context Free Grammars

Consider the following CFG which generates strings from the language V $:= \{0, 1, 2, 3, 4\}^*$

$$\textbf{S} \rightarrow 0\textbf{X}4$$
$$\textbf{X} \rightarrow 1\textbf{X}3 \mid 2$$

List 5 strings generated by the CFG and 5 strings from V not generated by the CFG. Then, summarize this CFG in your own words.

2. Constructing CFGs

For each of the following, construct a CFG for the specified language.

(a) Strings from the language $S := \{a\}^*$ with an even number of a's.

(b) Strings from the language $S := \{a, b\}^*$ with odd length.

(c) Strings from the language $S := \{a, b\}^*$ with an even number of a's or an odd number of b's.

(d) Strings from the language $S := \{a, b\}^*$ with an equal number of a's and b's.

3. Relations Examples

(a) Consider the relation $R \subseteq \mathbb{Z} \times \mathbb{Z}$ defined by $(a, b) \in R$ iff $a \leq b + 1$. List 3 pairs of integers that are in R, and 3 pairs of integers that are not.

(b) Consider the relation $R \subseteq \mathbb{Z} \times \mathbb{Z}$ defined by $(a, b) \in R$ iff $a \leq b+1$. Determine if R is reflexive, symmetric, antisymmetric, and/or transitive. If a relation has a property, explain why. If not, state a counterexample.

4. Relations Proofs

Suppose that $R, S \subseteq \mathbb{Z} \times \mathbb{Z}$ are relations.

(a) Prove or disprove: If R and S are transitive, $R \cup S$ is transitive.

(b) Prove or disprove: If R is symmetric, $\overline{R}$ (the complement of R) is symmetric.