Conceptual Review

Relations definitions: Let \( R \) be a relation on \( A \). In other words, \( R \subseteq A \times A \). Then:

- \( R \) is reflexive iff for all \( a \in A \), \((a, a) \in R\).
- \( R \) is symmetric iff for all \( a, b \), if \((a, b) \in R\), then \((b, a) \in R\).
- \( R \) is antisymmetric iff for all \( a, b \), if \((a, b) \in R\) and \( a \neq b \), then \((b, a) \notin R\).
- \( R \) is transitive iff for all \( a, b \), if \((a, b) \in R\) and \((b, c) \in R\), then \((a, c) \in R\).

1. Context Free Grammars

Consider the following CFG which generates strings from the language \( V := \{0, 1, 2, 3, 4\}^* \)

\[
S \rightarrow 0X4 \\
X \rightarrow 1X3 \mid 2
\]

List 5 strings generated by the CFG and 5 strings from \( V \) not generated by the CFG. Then, summarize this CFG in your own words.
2. Constructing CFGs
For each of the following, construct a CFG for the specified language.

(a) Strings from the language $S := \{a\}^*$ with an even number of $a$'s.

(b) Strings from the language $S := \{a, b\}^*$ with odd length.

(c) Strings from the language $S := \{a, b\}^*$ with an even number of $a$'s or an odd number of $b$'s.

(d) Strings from the language $S := \{a, b\}^*$ with an equal number of $a$'s and $b$'s.
3. Relations Examples
(a) Consider the relation \( R \subseteq \mathbb{Z} \times \mathbb{Z} \) defined by \((a, b) \in R \iff a \leq b + 1\). List 3 pairs of integers that are in \( R \), and 3 pairs of integers that are not.

(b) Consider the relation \( R \subseteq \mathbb{Z} \times \mathbb{Z} \) defined by \((a, b) \in R \iff a \leq b + 1\). Determine if \( R \) is reflexive, symmetric, antisymmetric, and/or transitive. If a relation has a property, explain why. If not, state a counterexample.

4. Relations Proofs
Suppose that \( R, S \subseteq \mathbb{Z} \times \mathbb{Z} \) are relations.

(a) Prove or disprove: If \( R \) and \( S \) are transitive, \( R \cup S \) is transitive.

(b) Prove or disprove: If \( R \) is symmetric, \( \overline{R} \) (the complement of \( R \)) is symmetric.