CSE 390Z: Mathematics for Computation Workshop

Week 4 Workshop

Conceptual Review

(a) Translate "all cats are friends with a dog" to predicate logic. Domain of dicourse: mammals.

(b) Inference Rules:

Introduce $\vee$:	$\frac{A}{\therefore A \vee B,\ B \vee A}$	Eliminate $\vee$:	$\frac{A \vee B\ ;\ \neg A}{\therefore B}$
Introduce $\wedge$:	$\frac{A\ ;\ B}{\therefore A \wedge B}$	Eliminate $\wedge$:	$\frac{A \wedge B}{\therefore A\ ,\ B}$
Direct Proof:	$\frac{A \Rightarrow B}{\therefore A \rightarrow B}$	Modus Ponens:	$\frac{A\ ;\ A \rightarrow B}{\therefore B}$
Intro $\exists$:	$\frac{P(c) \text{ for some } c}{\therefore \exists x P(x)}$	Eliminate $\exists$:	$\frac{\exists x P(x)}{\therefore P(c) \text{ for a fresh } c}$
Intro $\forall$:	$\frac{P(a);\ a \text{ is arbitrary}}{\therefore \forall x P(x)}$	Eliminate $\forall$:	$\frac{\forall x P(x)}{\therefore P(a);\ a \text{ is arbitrary}}$

(c) What are DeMorgan's Laws for Quantifiers?

(d) Given $A \wedge B$, prove $A \vee B$

Given $P \rightarrow R$, $R \rightarrow S$, prove $P \rightarrow S$.

(e) How do you prove a "for all" statement? E.g. prove $\forall x P(x)$. How do you prove a "there exists" statement? E.g. prove $\exists x P(x)$.

1. Tricky Translations

Translate the following English sentences to predicate logic. The domain is integers, and you may use $=$, $\neq$, and $>$ as predicates. Assume the predicates Prime, Composite, and Even have been defined appropriately.

(a) 2 is prime.

(b) Every **positive** integer is prime or composite, but not both.

(c) There is **exactly one** even prime.

(d) 2 is the only even prime.

(e) Some, but not all, composite integers are even.

2. Propositional Proof 1

(a) Prove that given $p \rightarrow q$, $\neg s \rightarrow \neg q$, and p, we can conclude s.

(b) Prove that given $\neg(p \vee q) \to s$, $\neg p$, and $\neg s$, we can conclude q.

3. Propositional Proofs 2

(a) Prove that given $p \to q$, we can conclude $(p \wedge r) \to q$

(b) Prove that given $p \vee q$, $q \to r$, and $r \to s$, we can conclude $\neg p \to s$.

4. Predicate Proofs 1

(a) Prove that $\forall x P(x) \to \exists x P(x)$. You may assume that the domain is nonempty.

(b) Given $\forall x(T(x) \to M(x))$ and $\exists x(T(x))$, prove that $\exists x(M(x))$.

(c) Given $\forall x(P(x) \to Q(x))$, prove that $\exists x P(x) \to \exists y Q(y)$. You may assume that the domain is non-empty.

5. Negating Predicates

Last week, we translated the sentence "Not every actor has been featured in a movie" to predicate logic. The domain of discourse was movies and actors, and we had the following predicates: Movie(x) $::=$ x is a movie, Actor(x) $::=$ x is an actor, Features(x, y) $::=$ x features y.

This was Timothy's translation: $\neg\forall x(\text{Actor}(x) \rightarrow \exists y(\text{Movie}(y) \wedge \text{Features}(y, x)))$

This was Cade's translation: $\exists x(\text{Actor}(x) \wedge \forall y(\text{Movie}(y) \rightarrow \neg\text{Features}(y, x)))$

(a) Adam claims that Timothy and Cade are both correct. Do you agree with Adam?

(b) Use a chain of predicate logic equivalences to prove that Timothy and Cade's translations are equivalent. **Hint:** You may wish to use De Morgan's Law for Predicates and the Law of Implications.

6. Predicate Proofs 2

Given $\forall x\ (P(x) \vee Q(x))$ and $\forall y\ (\neg Q(y) \vee R(y))$, prove $\exists x\ (P(x) \vee R(x))$. You may assume that the domain is not empty.

7. Predicate Proofs 3

Write a formal proof to show: If n, m are odd, then $n + m$ is even.

Let the predicates $\mathsf{Odd}(x)$ and $\mathsf{Even}(x)$ be defined as follows where the domain of discourse is integers:

$$\mathsf{Odd}(x) := \exists y \ (x = 2y + 1)$$
$$\mathsf{Even}(x) := \exists y \ (x = 2y)$$

8. Challenge: Predicate Negation

Translate "You can fool all of the people some of the time, and you can fool some of the people all of the time, but you can't fool all of the people all of the time" into predicate logic. Then, negate your translation. Then, translate the negation back into English.

Hint: Let the domain of discourse be all people and all times, and let $P(x, y)$ be the statement "You can fool person x at time y". You can get away with not defining any other predicates if you use P.

9. Challenge: Formal Proof

Use a formal proof to show that for any propositions a, b, c, the following holds.

$$a \rightarrow (b \rightarrow (c \rightarrow ((a \wedge b) \wedge c)))$$