CSE 390Z: Mathematics for Computation Workshop

Practice 311 Midterm 1
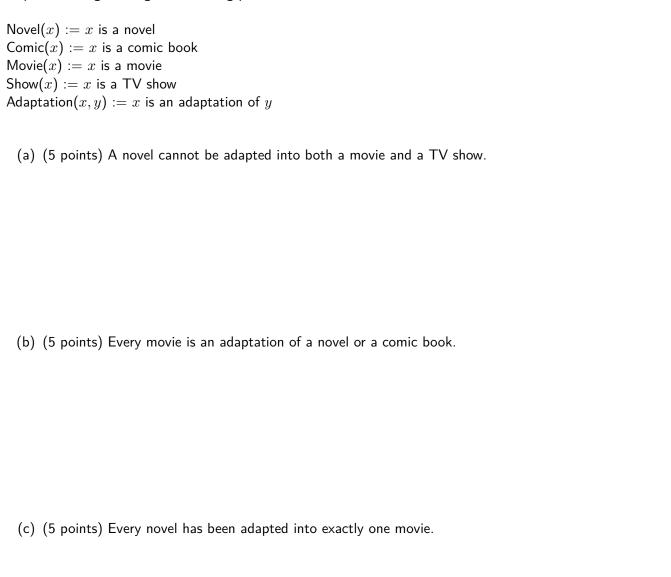
Name: _____

UW ID: _____

Instructions:

- This is a **simulated practice midterm**. You will **not** be graded on your performance on this exam.

- Nevertheless, please treat this as if it is a real exam. That means that you may not discuss with your neighbors, reference outside material, or use your devices during the next 50 minute period.

- If you get stuck on a problem, consider moving on and coming back later. In the actual exam, there will likely be opportunity for partial credit.

- There are 3 problems on this exam, totaling 55 points.

1. Predicate Translation [15 points]

Let the domain of discourse be novels, comic books, movies, and TV shows. Translate the following statements to predicate logic, using the following predicates:

$\text{Novel}(x) := x$ is a novel
$\text{Comic}(x) := x$ is a comic book
$\text{Movie}(x) := x$ is a movie
$\text{Show}(x) := x$ is a TV show
$\text{Adaptation}(x, y) := x$ is an adaptation of y

(a) (5 points) A novel cannot be adapted into both a movie and a TV show.

(b) (5 points) Every movie is an adaptation of a novel or a comic book.

(c) (5 points) Every novel has been adapted into exactly one movie.

2. Number Theory Proof [20 points]

Recall this definition of odd: $\text{Odd}(x) := \exists y(x = 2y + 1)$. Write an English proof to show that for all odd integers k, the statement $8 \mid k^2 - 1$ holds.

Hint: At some point in your proof, you'll need to show that for any integer a, $a(a+1)$ is even. When you reach this point, feel free to break your proof up into the case where a is even, and the case where a is odd.

3. Induction [20 points]

Prove by induction that $3^n - 1$ is divisible by 2 for any integer $n \geq 1$.