CSE 390Z: Mathematics for Computation Workshop

QuickCheck: Induction Proof Solutions (due Monday, February 13)

Please submit a response to the following questions on Gradescope. We do not grade on accuracy, so please submit your best attempt. You may either typeset your responses or hand-write them. Note that hand-written solutions must be legible to be graded.

We have created **this template** if you choose to typeset with Latex. **This guide** has specific information about scanning and uploading pdf files to Gradescope.

0. Induction Junction, what's your function?

The sum of integers up to n can be represented as $0 + 1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$, where $n \in \mathbb{N}$ (this fact can actually be proven using induction).

Prove the following equality for all $n \in \mathbb{N}$

$$(0 + 1 + 2 + ... + n)^2 = 1^3 + 2^3 + ... + n^3$$

Hint: If the sum of integers up to n equals $\frac{n(n+1)}{2}$, then how would you represent the sum of integers up to n, squared? What about the sum of integers up to $n + 1$?

Solution:

Let $P(n)$ be the statement:

$$0^3 + 1^3 + 2^3 + ... + n^3 = (0 + 1 + 2 + ... + n)^2$$

We will prove that $P(n)$ holds for all $n \in N$ by induction on n.

Base Case $P(0)$: $0^3 = 0^2$, so $P(0)$ holds.

Inductive Hypothesis: Suppose that $P(k)$ is true for some arbitrary $k \in N$.

Inductive Step: We will show $P(k + 1)$ holds.

$$
\begin{aligned}
0^3 + 1^3 + ... + k^3 + (k+1)^3 &= (0^3 + 1^3 + ... + k^3) + (k+1)^3 && \text{[Associativity]} \\
&= (0 + 1 + ... + k)^2 + (k+1)^3 && \text{[Inductive Hypothesis]} \\
&= \left(\frac{k(k+1}{2}\right)^2 + (k+1)^3 && \text{[by given equivalence]} \\
&= (k+1)^2 \left(\frac{k^2}{4} + (k+1)\right) && \text{[Factor } (k+1)^2\text{]} \\
&= (k+1)^2 \left(\frac{k^2 + 4k + 4}{4}\right) && \text{[Add via common denominator]} \\
&= (k+1)^2 \left(\frac{(k+2)^2}{4}\right) && \text{[Factor numerator]} \\
&= \left(\frac{(k+1)^2(k+2)^2}{4}\right) && \text{[Algebra]} \\
&= \left(\frac{(k+1)(k+2)}{2}\right)^2 && \text{[Take out the square]} \\
&= (0 + 1 + ... + k + (k+1))^2 && \text{[by given equivalence]}
\end{aligned}
$$

And thus $P(k+1)$ holds for an arbitrary $k \in N$.

Conclusion: We have shown that $P(n)$ holds for all $n \in N$

1. Video Solution

Watch **this video** by 390z TA Timothy on the solution **after** making an initial attempt. Then, answer the following questions.

(a) What is one thing you took away from the video solution?

(b) What topic from the quick check or lecture would you most like to review in workshop?