0. Notes on Conceptual Review
Solution:
See slides for conceptual review.

1. RegEx, CFGs, and DFAs
Let \( \Sigma = \{0, 1, 2\} \). Consider the language "all strings with an even number of 2's."

(a) Design a regular expression for this language.

Solution:
\[(0 \cup 1 \cup (2(0 \cup 1)^* 2))^*\]

(b) Design a CFG for this language.

Solution:
\[S \rightarrow \varepsilon \mid 0S \mid 1S \mid S2S2S\]

(c) Design a DFA for this language.

Solution:

2. Constructing Regular Expressions
For each of the following, construct a regular expression for the specified language.

(a) Strings from the language \( \Sigma := \{a\}^* \) with an even number of \( a \)'s.

Solution:
\[(aa)^*\]

(b) Strings from the language \( \Sigma := \{a, b\}^* \) with an even number of \( a \)'s.

Solution:
\[b^*(b^*ab^*ab^*)^*\]

(c) Strings from the language \( \Sigma := \{a, b\}^* \) with odd length.
Solution:  

\[(aa \cup ab \cup ba \cup bb)(a \cup b)\]

(d) (Challenge) Strings from the language \(\Sigma := \{a, b\}^*\) with an even number of \(a\)'s or an odd number of \(b\)'s.

Solution:  

\[b^*(b^*ab^*ab^*)^* \cup (a^* \cup a^*ba^*ba^*)^*b(a^* \cup a^*ba^*ba^*)^*\]

3. Context Free Grammars

Consider the following CFG which generates strings from the language \(V := \{0, 1, 2, 3, 4\}^*\)

\[S \rightarrow 0X4\]
\[X \rightarrow 1X3 \mid 2\]

List 5 strings generated by the CFG and 5 strings from \(V\) not generated by the CFG. Then, summarize this CFG in your own words.

Solution:

Accepted:
- 024
- 01234
- 0112334
- 011123334
- 0111123334

Rejected:
- \(\epsilon\)
- 2
- 0244
- 011234
- 10234

This CFG is all strings of the form \(0 \ 1^m \ 2 \ 3^m \ 4\), where \(m \geq 0\). That is, it’s all strings made of one 0, followed by zero or more 1’s, followed by a 2, followed by the same number of 3’s as 1’s, followed by one 4.

4. Constructing CFGs

For each of the following, construct a CFG for the specified language.

(a) Strings from the language \(\Sigma := \{a\}^*\) with an even number of \(a\)'s.

Solution:

\[S \rightarrow aaS | \epsilon\]

(b) Strings from the language \(\Sigma := \{a, b\}^*\) with odd length.

Solution:

\[S \rightarrow CS | a | b\]
\[C \rightarrow aaC | abC | baC | bbC | \epsilon\]

(c) Strings from the language \(\Sigma := \{a, b\}^*\) with an even number of \(a\)'s or an odd number of \(b\)'s.
(d) Strings from the language $\Sigma := \{a, b\}^*$ with an equal number of $a$'s and $b$'s.

Solution:

$$S \rightarrow aSbS|bSaS|\varepsilon$$

5. Constructing DFAs

For each of the following, construct a DFA for the specified language.

(a) Strings from the language $\Sigma := \{a\}^*$ with an even number of $a$'s.

Solution:

(b) Strings from the language $\Sigma = \{a, b\}$ with an even number of $a$'s or an odd number of $b$'s.

Solution:

(c) Strings from the language $\Sigma = \{a, b\}$ with odd length.
### Solution:

\[
\begin{tikzpicture}
\node[circle, draw] (start) at (0,0) {even};
\node[circle, draw, right of=start] (end) {odd};
\draw[->] (start) -- node[above] {a,b} (end);
\draw[->] (end) -- node[above] {a,b} (start);
\end{tikzpicture}
\]

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### 6. Challenge: All the Machines!

Using the alphabet \( \Sigma = \{0, 1, 2, 3, 4, 5\} \), define the language \( L \) as follows. If \( x \) is a string from \( \Sigma^* \) with characters \( x_0, \ldots, x_n \), then \( x \in L \) iff: for every \( i \) between 0 and \( n \), if \( x_i \) is an odd digit, then \( x_k > x_i \) for every \( k > i \). For example, if one of the digits is a 3, ever digit after it must be a 4 or higher.

(a) List 3 strings in \( L \) and 3 strings from \( \Sigma^* \) not in \( L \).

**Solution:**

**Accepted:**
- 145
- 135
- 12425
- 2004
- 2034

**Rejected:**
- 321
- 11
- 455
- 452
- 2010

(b) Construct a regular expression for the language \( L \).

**Solution:**

\[
(0 \cup 2 \cup 4)^* (\varepsilon \cup 1) (2 \cup 4)^* (\varepsilon \cup 3) 4^* (\varepsilon \cup 5)
\]

(c) Construct a CFG for the language \( L \).

**Solution:**

\[
\begin{align*}
S &\rightarrow 0S | 2S | 4S | A \\
A &\rightarrow 1B | B \\
B &\rightarrow 2B | 4B | C \\
C &\rightarrow 3D | D \\
D &\rightarrow 4D | E \\
E &\rightarrow 5 | \varepsilon
\end{align*}
\]

(d) Construct a DFA for the language \( L \).
Solution: