CSE 390Z: Mathematics of Computing

Week 8 Workshop Solutions

0. Notes on Conceptual Review

Solution:

See slides for conceptual review.

1. RegEx, CFGs, and DFAs

Let $\Sigma = \{0, 1, 2\}$. Consider the language "all strings with an even number of 2's."

(a) Design a regular expression for this language..

Solution:

$(0 \cup 1 \cup (2(0 \cup 1)^*2))^*$

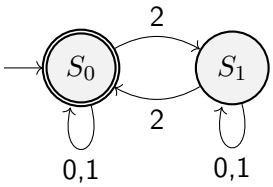
(b) Design a CFG for this language.

Solution:

$S \to \varepsilon \mid 0S \mid 1S \mid S2S2S$

(c) Design a DFA for this language.

Solution:



2. Constructing Regular Expressions

For each of the following, construct a regular expression for the specified language.

(a) Strings from the language $\Sigma := \{a\}^*$ with an even number of a's.

Solution:

$(aa)^*$

(b) Strings from the language $\Sigma := \{a, b\}^*$ with an even number of a's.

Solution:

$b^*(b^*ab^*ab^*)^*$

(c) Strings from the language $\Sigma := \{a, b\}^*$ with odd length.

Solution:

$(aa \cup ab \cup ba \cup bb)^*(a \cup b)$

(d) (Challenge) Strings from the language $\Sigma := \{a, b\}^*$ with an even number of a's or an odd number of b's.

Solution:

$b^*(b^*ab^*ab^*)^* \cup (a^* \cup a^*ba^*ba^*)^*b(a^* \cup a^*ba^*ba^*)^*$

3. Context Free Grammars

Consider the following CFG which generates strings from the language V $:= \{0, 1, 2, 3, 4\}^*$

$$\mathbf{S} \to 0\mathbf{X}4$$
$$\mathbf{X} \to 1\mathbf{X}3 \mid 2$$

List 5 strings generated by the CFG and 5 strings from V not generated by the CFG. Then, summarize this CFG in your own words.

Solution:

Accepted:

- 024
- 01234
- 0112334
- 011123334
- 01111233334

Rejected:

- ϵ
- 2
- 0244
- 011234
- 10234

This CFG is all strings of the form $0\ 1^m\ 2\ 3^m\ 4$, where $m \geq 0$. That is, it's all strings made of one 0, followed by zero or more 1's, followed by a 2, followed by the same number of 3's as 1's, followed by one 4.

4. Constructing CFGs

For each of the following, construct a CFG for the specified language.

(a) Strings from the language $\Sigma := \{a\}^*$ with an even number of a's.

Solution:

$\mathbf{S} \to aa\mathbf{S}|\varepsilon$

(b) Strings from the language $\Sigma := \{a, b\}^*$ with odd length.

Solution:

$$\mathbf{S} \to \mathbf{C}\mathbf{S}|a|b$$
$$\mathbf{C} \to aa\mathbf{C}|ab\mathbf{C}|ba\mathbf{C}|bb\mathbf{C}|\varepsilon$$

(c) Strings from the language $\Sigma := \{a, b\}^*$ with an even number of a's or an odd number of b's.

Solution:

$$\mathbf{S} \to \mathbf{E}|\mathbf{O}b\mathbf{O}$$
$$\mathbf{E} \to \mathbf{E}\mathbf{E}|a\mathbf{E}a|b|\varepsilon$$
$$\mathbf{O} \to \mathbf{O}\mathbf{O}|b\mathbf{O}b|a|\varepsilon$$

(d) Strings from the language $\Sigma := \{a, b\}^*$ with an equal number of a's and b's.

Solution:

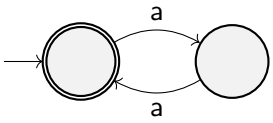
$$\mathbf{S} \to a\mathbf{S}b\mathbf{S}|b\mathbf{S}a\mathbf{S}|\varepsilon$$

5. Constructing DFAs

For each of the following, construct a DFA for the specified language.

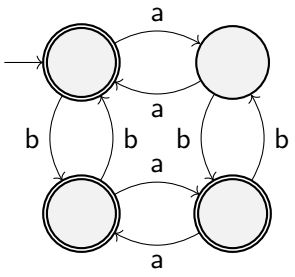
(a) Strings from the language $\Sigma := \{a\}^*$ with an even number of a's.

Solution:



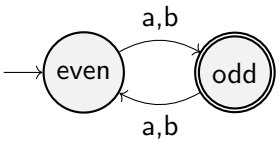
(b) Strings from the language $\Sigma = \{a, b\}$ with an even number of a's or an odd number of b's.

Solution:



(c) Strings from the language $\Sigma = \{a, b\}$ with odd length.

Solution:



6. Challenge: All the Machines!

Using the alphabet $\Sigma = \{0, 1, 2, 3, 4, 5\}$, define the language L as follows. If x is a string from $\Sigma*$ with characters $x_0, \ldots, x_n$, then $x \in L$ iff: for every i between 0 and n, if x_i is an odd digit, then $x_k > x_i$ for every $k > i$. For example, if one of the digits is a 3, ever digit after it must be a 4 or higher.

(a) List 3 strings in L and 3 strings from $\Sigma*$ not in L.

Solution:

Accepted:

- 145
- 135
- 12425
- 2004
- 2034

Rejected:

- 321
- 11
- 455
- 452
- 2010

(b) Construct a regular expression for the language L.

Solution:

$(0 \cup 2 \cup 4)^*(\varepsilon \cup 1)(2 \cup 4)^*(\varepsilon \cup 3)4^*(\varepsilon \cup 5)$

(c) Construct a CFG for the language L.

Solution:

$$\begin{aligned}
\mathbf{S} &\to 0\mathbf{S}|2\mathbf{S}|4\mathbf{S}|\mathbf{A} \\
\mathbf{A} &\to 1\mathbf{B}|\mathbf{B} \\
\mathbf{B} &\to 2\mathbf{B}|4\mathbf{B}|\mathbf{C} \\
\mathbf{C} &\to 3\mathbf{D}|\mathbf{D} \\
\mathbf{D} &\to 4\mathbf{D}|\mathbf{E} \\
\mathbf{E} &\to 5|\varepsilon
\end{aligned}$$

(d) Construct a DFA for the language L.

Solution: