CSE 390Z: Mathematics for Computation Workshop

Practice 311 Final

Name: ______________________________________

UW ID: ______________________________________

Instructions:

- This is a **simulated practice final**. You will **not** be graded on your performance on this exam.
- This final was written to take 50 minutes. The real final will be an hour and 50 minutes.
- Nevertheless, please treat this as if it is a real exam. That means that you may not discuss with your neighbors, reference outside material, or use your devices during the next 50 minute period.
- If you get stuck on a problem, consider moving on and coming back later. In the actual exam, there will likely be opportunity for partial credit.
- There are 5 problems on this exam.

1. All the Machines! [15 points]
Let the alphabet be $\Sigma = \{a, b\}$. Consider the language $L = \{w \in \Sigma^* : \text{every } a \text{ has a } b \text{ two characters later}\}$. In other words, L is the language of all strings in the alphabet a, b where after any a, the character after the a can be anything, but the character after that one must be a b.

Some strings in L include ε, abb, $aabb$, $bbbbabb$. Some strings not in L include a, ab, aab, $ababb$. Notice that the last two characters of the string cannot be an a.

(a) (5 points) Give a regular expression that represents L.

(b) (5 points) Give a CFG that represents L.

(c) (5 points) Give a DFA that represents L.

2. Induction 1 [20 points]
Recall the recursive definition of a list of integers:

- $[\,]$ is the empty list
- If L is a list and a is an integer, then $a :: L$ is a list whose first element is a, followed by the elements of L.

Consider the following functions defined on lists:
len($[\,]$) $= 0$
len($x :: L$) $= 1+$ len(L)

inc($[\,]$) $= [\,]$
inc($x :: L$) $= (x+1) ::$ inc(L)

sum($[\,]$) $= 0$
sum($x :: L$) $= x +$ sum(L)

Prove that for all lists L, sum(inc(L)) $=$ sum(L) $+$ len(L).

3. Induction 2 [20 points]
Consider the following recursive definition of a_n:

$$a_1 = 1$$
$$a_2 = 1$$
$$a_n = \frac{1}{2}(a_{n-1} + \frac{2}{a_{n-2}}) \qquad \text{for } n > 2$$

Prove that $1 \leq a_n \leq 2$ for all integers $n \geq 1$.

4. Modular Arithmetic [10 points]
(a) Prove or disprove: If $a \equiv b \pmod{10}$, then $a \equiv b \pmod{5}$. [5 points]

(b) Prove or disprove: If $a \equiv b \pmod{10}$, then $a \equiv b \pmod{20}$. [5 points]

5. Irregularity [20 points]
Prove that the set of strings $\{0^n10^n : n \geq 0\}$ is not regular.