CSE 390Z: Mathematics of Computing

Week 8 Workshop

0. Notes on Conceptual Review

1. RegEx, CFGs, and DFAs

Let $\Sigma = \{0, 1, 2\}$. Consider the language "all strings with an even number of 2's."

(a) Design a regular expression for this language..

(b) Design a CFG for this language.

(c) Design a DFA for this language.

2. Constructing Regular Expressions

For each of the following, construct a regular expression for the specified language.

(a) Strings from the language $\Sigma := \{a\}^*$ with an even number of a's.

(b) Strings from the language $\Sigma := \{a, b\}^*$ with an even number of a's.

(c) Strings from the language $\Sigma := \{a, b\}^*$ with odd length.

(d) (Challenge) Strings from the language $\Sigma := \{a, b\}^*$ with an even number of a's or an odd number of b's.

3. Context Free Grammars

Consider the following CFG which generates strings from the language V $:= \{0, 1, 2, 3, 4\}^*$

$$\mathbf{S} \to 0\mathbf{X}4$$
$$\mathbf{X} \to 1\mathbf{X}3 \mid 2$$

List 5 strings generated by the CFG and 5 strings from V not generated by the CFG. Then, summarize this CFG in your own words.

4. Constructing CFGs

For each of the following, construct a CFG for the specified language.

(a) Strings from the language $\Sigma := \{a\}^*$ with an even number of a's.

(b) Strings from the language $\Sigma := \{a, b\}^*$ with odd length.

(c) Strings from the language $\Sigma := \{a, b\}^*$ with an even number of a's or an odd number of b's.

(d) Strings from the language $\Sigma := \{a, b\}^*$ with an equal number of a's and b's.

5. Constructing DFAs

For each of the following, construct a DFA for the specified language.

(a) Strings from the language $\Sigma := \{a\}^*$ with an even number of a's.

(b) Strings from the language $\Sigma = \{a, b\}$ with an even number of a's or an odd number of b's.

(c) Strings from the language $\Sigma = \{a, b\}$ with odd length.

6. Challenge: All the Machines!

Using the alphabet $\Sigma = \{0, 1, 2, 3, 4, 5\}$, define the language L as follows. If x is a string from $\Sigma*$ with characters $x_0, \ldots, x_n$, then $x \in L$ iff: for every i between 0 and n, if x_i is an odd digit, then $x_k > x_i$ for every $k > i$. For example, if one of the digits is a 3, ever digit after it must be a 4 or higher.

(a) List 3 strings in L and 3 strings from $\Sigma*$ not in L.

(b) Construct a regular expression for the language L.

(c) Construct a CFG for the language L.

(d) Construct a DFA for the language L.