0. Induction: Divides
Prove that $9 \mid (n^3 + (n + 1)^3 + (n + 2)^3)$ for all $n > 1$ by induction.
1. **Induction: Equality**

For any $n \in \mathbb{N}$, define $S_n$ to be the sum of the squares of the first $n$ positive integers, or

$$S_n = 1^2 + 2^2 + \cdots + n^2.$$ 

Prove that for all $n \in \mathbb{N}$, $S_n = \frac{1}{6} n(n + 1)(2n + 1)$. 

2. Induction: Inequality

Prove by induction on $n$ that for all integers $n \geq 0$ the inequality $(3 + \pi)^n \geq 3^n + n\pi3^{n-1}$ is true.
3. Induction: Another Inequality

Prove by induction on $n$ that for all integers $n \geq 4$ the inequality $n! > 2^n$ is true.
4. **Strong Induction: Stamp Collection**

A store sells 3 cent and 5 cent stamps. Use strong induction to prove that you can make exactly $n$ cents worth of stamps for all $n \geq 10$.

**Hint:** you’ll need multiple base cases for this - think about how many steps back you need to go for your inductive step.
5. **Strong Induction: Functions**

Consider the function $f(n)$ defined for integers $n \geq 1$ as follows:

$f(1) = 1$ for $n = 1$

$f(2) = 4$ for $n = 2$

$f(3) = 9$ for $n = 3$

$f(n) = f(n - 1) - f(n - 2) + f(n - 3) + 2(2n - 3)$ for $n \geq 4$

Prove by strong induction that for all $n \geq 1$, $f(n) = n^2$. 
6. **Strong Induction: Collecting Candy**

A store sells candy in packs of 4 and packs of 7. Let $P(n)$ be defined as "You are able to buy $n$ packs of candy". For example, $P(3)$ is not true, because you cannot buy exactly 3 packs of candy from the store. However, it turns out that $P(n)$ is true for any $n \geq 18$. Use strong induction on $n$ to prove this.

**Hint:** you’ll need multiple base cases for this - think about how many steps back you need to go for your inductive step.