CSE 390Z: Mathematics for Computation Workshop

Week 2 Workshop Problems

Conceptual Review

(a) Fill in the following definitions.

Tautology:

Contradiction:

Contingency:

(b) What is the contrapositive of $p \rightarrow q$? What is the converse of $p \rightarrow q$?

Contrapositive:

Converse:

(c) What are two different methods to show that two propositions are equivalent?

(d) To prove a chain of equivalences, there are many rules we can use (attached to the back of this handout). Fill in some of those rules here.

DeMorgan's Law:

Law of Implication:

Contrapositive:

(e) What's the difference between propositional logic, boolean algebra, and circuits?

1. Translation: Running from my problems

Define a set of three atomic propositions, and use them to translate the following sentences.

(i) Whenever it's snowing and it's Friday, I am not going for a run.

(ii) I am going for a run because it is not snowing.

(iii) I am going for a run only if it is not Friday or not snowing.

2. Translation: Age is just a number

Define a set of two atomic propositions, and use them to translate the following sentences.

(i) If Kai is older than thirty, then Kai is older than twenty.

(ii) Kai is older than thirty only if Kai is older than twenty.

(iii) Whenever Kai is older than thirty, Kai is older than twenty.

(iv) Kai being older than twenty is necessary for Kai to be older than thirty.

3. Truth Table

Draw a truth table for $(p \to \neg q) \to (r \oplus q)$

4. Equivalences: Propositional Logic

Write a chain of logical equivalences to prove the following statements. Note that with propositional logic, you are expected to show all steps, including commutativity and associativity.

(a) $p \to q \equiv \neg(p \land \neg q)$

(b) $\neg p \lor ((q \land p) \lor (\neg q \land p)) \equiv T$

(c) $((p \wedge q) \rightarrow r) \equiv (p \rightarrow r) \vee (q \rightarrow r)$

5. Equivalences: Boolean Algebra

(a) Prove $p' + (p \cdot q) + (q' \cdot p) = 1$ via equivalences.

(b) Prove $(p' + q) \cdot (q + p) = q$ via equivalences.

6. Implications and Vacuous Truth

Alice and Bob's teacher says in class "if a number is prime, then the number is odd." Alice and Bob both believe that the teacher is wrong, but for different reasons.

(a) Alice says "9 is odd and not prime, so the implication is false." Is Alice's justification correct? Why or why not?

(b) Bob says "2 is prime and not odd, so the implication is false." Is Bob's justification correct? Why or why not?

(c) Recall that this is the truth table for implications. Which row does Alice's example correspond to? Which row does Bob's example correspond to?

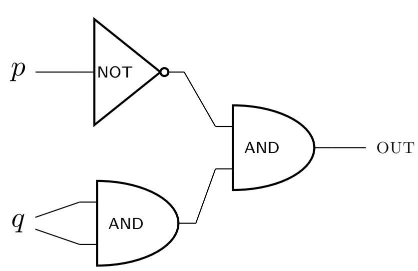
p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

(d) Observe that in order to show that $p \to q$ is false, you need an example where p is true and q is false. Examples where p is false don't disprove the implication! (Nothing to write for this part).

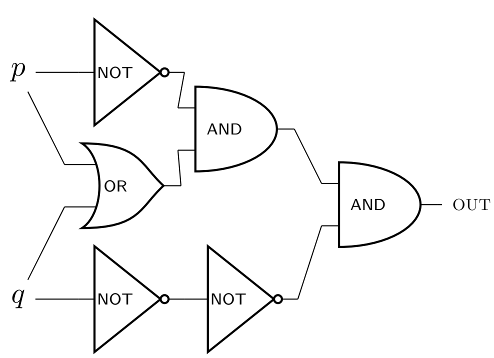
7. Circuits

Convert the following ciruits into logical expressions.

(i)



(ii)



8. Boolean Algebra

Which of the following boolean algebra expressions are equivalent?

(1) $((a' + b') \cdot (a + b))' + (a \cdot b')$

(2) a

(3) b

(4) $a + b'$

(5) $(a' \cdot b)'$

(6) $(a' + b') \cdot a$

(7) $(a \cdot b) + b'$