CSE 390Z: Mathematics for Computation Workshop

QuickCheck: Induction Solutions

Please submit a response to the following questions on Gradescope. We do not grade on accuracy, so please submit your best attempt. You may either typeset your responses or hand-write them. Note that hand-written solutions must be legible to be graded.

We have created this template if you choose to typeset with Latex. This guide has specific information about scanning and uploading pdf files to Gradescope.

0. Induction

Prove by induction that for all integers \( n \geq 1 \):

\[
\sum_{i=1}^{n} 5i = \frac{5n(n+1)}{2}
\]

Solution:

1. Let \( P(n) \) be the statement \( \sum_{i=1}^{n} 5i = \frac{5n(n+1)}{2} \). We prove \( P(n) \) for all integers \( n \geq 1 \) by induction.

2. Base Case: When \( n = 1 \), the left-hand side is \( 5 \times 1 = 5 \). The right-hand side is \( \frac{5n(n+1)}{2} = \frac{5(1+1)}{2} = \frac{5 \times 2}{2} = 5 \). Since 5 = 5, the base case holds.

3. Inductive Hypothesis: Suppose that \( P(k) \) holds for some arbitrary integer \( k \geq 1 \). Then \( \sum_{i=1}^{k} 5i = \frac{5k(k+1)}{2} \).

4. Inductive Step: Observe that...

\[
\sum_{i=1}^{k+1} 5i = \left( \sum_{i=1}^{k} 5i \right) + 5(k+1)
\]

By Definition of Sum

\[
= \frac{5k(k+1)}{2} + 5(k+1)
\]

By IH

\[
= \frac{5k(k+1)}{2} + \frac{10(k+1)}{2}
\]

Algebra

\[
= \frac{5k(k+1) + 10(k+1)}{2}
\]

Algebra

\[
= \frac{(5k+10)(k+1)}{2}
\]

Algebra

\[
= \frac{5(k+2)(k+1)}{2}
\]

Algebra

\[
= \frac{5(k+1)(k+2)}{2}
\]

Algebra

Thus \( \sum_{i=1}^{k+1} 5i = \frac{5(k+1)(k+2)}{2} \). So \( P(k+1) \) holds.

5. Thus we have proven \( P(n) \) for all integers \( n \geq 1 \) by induction.
1. Video Solution
Watch this solution video after making an initial attempt. Then, answer the following questions.

(a) What is one thing you took away from the video solution?

(b) On Tuesday, May 9th, we will do a full simulated 311 midterm in workshop. What is one thing you plan to do, or have already done, to prepare for the 311 midterm?