Week 2 Review

Proposition: True or False statement

E.g. " The cat walks whenever it rains."

- Tautology: Always True. E.g $p \vee \neg p$.
- Contradiction: Always False. E.g. $p \wedge \neg p$.
- Contingency: sometimes true, sometimes false. E.g. $p \vee q$

An implication is written $p \to q$.

E.g. "If it's raining, I have my umbrella." Let $p :=$ it's raining, $q :=$ I have my umbrella. Then the implication would be translated to $p \to q$.

- The contrapositive of $p \to q$ is $\neg q \to \neg p$.

  E.g. "If I don't have my umbrella, it is not raining."

  Interestingly, the implication is equivalent to it's contrapositive. That is, if I know $p \to q$, I know $\neg q \to \neg p$ and vice versa.

- The converse of $p \to q$ is $q \to p$.

  E.g. "If I have my umbrella, it is raining."

  The implication is NOT equivalent to its converse.

  E.g. "If a shape is a square, it is a rectangle." is true, but "If a shape is a rectangle, it is a square." is not true.

We can use a variety of techniques to show that two propositions are equivalent.

① Draw a truth table for both propositions.

If every row has the same truth value, they are equivalent.

p	q	$p \to q$	$\neg p \vee q$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

$p \to q \equiv \neg p \vee q$

② Use known rules to write a chain of equivalences from one proposition to the other.

$\neg q \to p \equiv \neg\neg q \vee p$	Law of Implication
$\equiv q \vee p$	Double Negation
$\equiv p \vee q$	~~Associativity~~ Commutativity
$\equiv \neg\neg p \vee q$	Double Negation
$\equiv \neg p \to q$	Law of Implication

$\neg q \to p \equiv \neg p \to q$

DeMorgan's Laws: $\neg(p \vee q) \equiv \neg p \wedge \neg q$

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

Law of Implication: $p \to q \equiv \neg p \vee q$

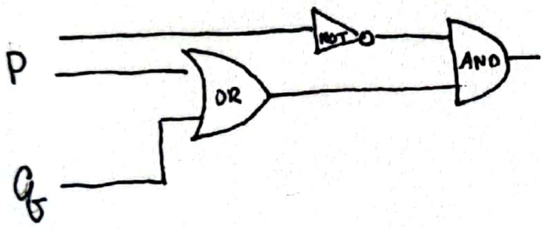
Contrapositive: $p \to q \equiv \neg q \to \neg p$

In class, we learned about propositional logic, circuits, & boolean algebra.

↳ These are notation to convey the same underlying meaning.

$(p \vee q) \wedge \neg p$ Propositional Logic

$(p + q) \cdot p'$ Boolean Algebra



Circuits