CSE 390Z: Mathematics of Computing Workshop

Week 9 Workshop

0. Conceptual Review

(a) Regular expression rules:
Basis: ϵ, $\varnothing$, a for $a \in \Sigma$
Recursive: If A, B are regular expressions, $(A \cup B), AB$, and A^* are regular expressions.

(b) Match the relation property to its definition:

Reflexive	$\forall a, b((a,b) \in R \wedge (b,c) \in R) \rightarrow (a,c) \in R)$
Symmetric	$\forall a(a \in A \rightarrow (a,a) \in R)$
Antisymmetric	$\forall a, b((a,b) \in R \rightarrow (b,a) \in R)$
Transitive	$\forall a, b(((a,b) \in R \wedge a \neq b) \rightarrow (b,a) \notin R)$.

1. Regular Expressions Warmup

Consider the following Regular Expression (RegEx):

$$1(45 \cup 54)^{\star}1$$

List 5 strings accepted by the RegEx and 5 strings from $T := \{1, 4, 5\}^{\star}$ rejected by the RegEx. Then, summarize this RegEx in your own words.

2. Context Free Grammars Warmup

Consider the following CFG which generates strings from the language V $:= \{0, 1, 2, 3, 4\}^*$

$$\mathbf{S} \to 0\mathbf{X}4$$
$$\mathbf{X} \to 1\mathbf{X}3 \mid 2$$

List 5 strings generated by the CFG and 5 strings from V not generated by the CFG. Then, summarize this CFG in your own words.

3. Simplify the RegEx

Consider the following Regular Expression (RegEx):

$$0^\star(0 \cup 1)^\star((01) \cup (11) \cup (10) \cup (00))1^\star(0 \cup 1)^\star$$

List 3 strings accepted by the RegEx and 3 strings from $S := \{0, 1\}^\star$ rejected by the RegEx. Then, summarize this RegEx in your own words and write a simpler RegEx that accepts exactly the same set of strings.

4. Constructing RegExs and CFGs

For each of the following, construct a regular expression and CFG for the specified language.

(a) Strings from the language $S := \{a\}^*$ with an even number of a's.

(b) Strings from the language $S := \{a, b\}^*$ with an even number of a's.

(c) Strings from the language $S := \{a, b\}^*$ with odd length.

(d) (Challenge) Strings from the language $S := \{a, b\}^*$ with an even number of a's or an odd number of b's.

5. Relations Examples

(a) Consider the relation $R \subseteq \mathbb{Z} \times \mathbb{Z}$ defined by $(a, b) \in R$ iff $a \leq b + 1$. List 3 pairs of integers that are in R, and 3 pairs of integers that are not.

(b) Consider the relation $R \subseteq \mathbb{Z} \times \mathbb{Z}$ defined by $(a, b) \in R$ iff $a \leq b+1$. Determine if R is reflexive, symmetric, antisymmetric, and/or transitive. If a relation has a property, explain why. If not, state a counterexample.

6. Relations Proofs

Suppose that $R, S \subseteq \mathbb{Z} \times \mathbb{Z}$ are relations.

(a) Prove or disprove: If R and S are transitive, $R \cup S$ is transitive.

(b) Prove or disprove: If R is symmetric, $\overline{R}$ (the complement of R) is symmetric.

7. Constructing DFAs

For each of the following, construct a DFA for the specified language.

(a) Strings from the language $\Sigma := \{a\}^*$ with an even number of a's.

(b) Strings from the language $\Sigma = \{a, b\}$ with odd length.

(c) Strings from the language $\Sigma = \{a, b\}$ with an even number of a's or an odd number of b's.