0. Conceptual Review
(a) Regular expression rules:
Basis: \( \epsilon, \emptyset, a \) for \( a \in \Sigma \)
Recursive: If \( A, B \) are regular expressions, \( (A \cup B), AB, \) and \( A^* \) are regular expressions.

(b) Match the relation property to its definition:
- Reflexive \( \forall a, b((a, b) \in R \land (b, c) \in R) \rightarrow (a, c) \in R \)
- Symmetric \( \forall a(a \in A \rightarrow (a, a) \in R) \)
- Antisymmetric \( \forall a, b((a, b) \in R \rightarrow (b, a) \notin R) \)
- Transitive \( \forall a, b(((a, b) \in R \land a \neq b) \rightarrow (b, a) \notin R) \).

1. Regular Expressions Warmup
Consider the following Regular Expression (RegEx):

\[ 1(45 \cup 54)^*1 \]

List 5 strings accepted by the RegEx and 5 strings from \( T := \{1, 4, 5\}^* \) rejected by the RegEx. Then, summarize this RegEx in your own words.
2. Context Free Grammars Warmup
Consider the following CFG which generates strings from the language $V := \{0, 1, 2, 3, 4\}^*$

$$S \rightarrow 0X4$$
$$X \rightarrow 1X3 \mid 2$$

List 5 strings generated by the CFG and 5 strings from $V$ not generated by the CFG. Then, summarize this CFG in your own words.

3. Simplify the RegEx
Consider the following Regular Expression (RegEx):

$0^*(0 \cup 1)^*(((01) \cup (11) \cup (10) \cup (00)))1^*(0 \cup 1)^*$

List 3 strings accepted by the RegEx and 3 strings from $S := \{0, 1\}^*$ rejected by the RegEx. Then, summarize this RegEx in your own words and write a simpler RegEx that accepts exactly the same set of strings.
4. Constructing RegExs and CFGs

For each of the following, construct a regular expression and CFG for the specified language.

(a) Strings from the language \( S := \{a\}^* \) with an even number of \( a \)'s.

(b) Strings from the language \( S := \{a, b\}^* \) with an even number of \( a \)'s.

(c) Strings from the language \( S := \{a, b\}^* \) with odd length.

(d) (Challenge) Strings from the language \( S := \{a, b\}^* \) with an even number of \( a \)'s or an odd number of \( b \)'s.
5. Relations Examples
(a) Consider the relation \( R \subseteq \mathbb{Z} \times \mathbb{Z} \) defined by \((a, b) \in R \text{ iff } a \leq b + 1\). List 3 pairs of integers that are in \( R \), and 3 pairs of integers that are not.

(b) Consider the relation \( R \subseteq \mathbb{Z} \times \mathbb{Z} \) defined by \((a, b) \in R \text{ iff } a \leq b + 1\). Determine if \( R \) is reflexive, symmetric, antisymmetric, and/or transitive. If a relation has a property, explain why. If not, state a counterexample.

6. Relations Proofs
Suppose that \( R, S \subseteq \mathbb{Z} \times \mathbb{Z} \) are relations.

(a) Prove or disprove: If \( R \) and \( S \) are transitive, \( R \cup S \) is transitive.

(b) Prove or disprove: If \( R \) is symmetric, \( \overline{R} \) (the complement of \( R \)) is symmetric.
7. Constructing DFAs
For each of the following, construct a DFA for the specified language.
(a) Strings from the language \( \Sigma := \{a\}^* \) with an even number of \( a \)'s.
(b) Strings from the language \( \Sigma = \{a, b\} \) with odd length.
(c) Strings from the language \( \Sigma = \{a, b\} \) with an even number of \( a \)'s or an odd number of \( b \)'s.