CSE 390Z: Mathematics for Computation Workshop

Week 4 Workshop

Conceptual Review

Set Theory

(a) **Definitions**
Set Equality: $A = B := \forall x(x \in A \leftrightarrow x \in B)$
Subset: $A \subseteq B := \forall x(x \in A \rightarrow x \in B)$
Union: $A \cup B := \{x \ : \ x \in A \vee x \in B\}$
Intersection: $A \cap B := \{x \ : \ x \in A \wedge x \in B\}$
Set Difference: $A \setminus B = A - B := \{x \ : \ x \in A \wedge x \notin B\}$
Set Complement: $\overline{A} = A^C := \{x \ : \ x \notin A\}$
Powerset: $\mathcal{P}(A) := \{B \ : B \subseteq A\}$
Cartesian Product: $A \times B := \{(a, b) \ : a \in A, \ b \in B\}$

(b) How do we prove that for sets A and B, $A \subseteq B$?

(c) How do we prove that for sets A and B, $A = B$?

Number Theory

(d) **Definitions**
a divides b: $a \mid b \quad \leftrightarrow \quad \exists k \in \mathbb{Z} \ (b = ka)$
a is congruent to b modulo m: $a \equiv b \pmod{m} \quad \leftrightarrow \quad m \mid (a - b)$

(e) What's the Division Theorem?

Set Theory

1. Set Operations

Let $A = \{1, 2, 5, 6, 8\}$ and $B = \{2, 3, 5\}$.

(a) What is the set $A \cap (B \cup \{2, 8\})$?

(b) What is the set $\{10\} \cup (A \setminus B)$?

(c) What is the set $\mathcal{P}(B)$?

(d) How many elements are in the set $A \times B$? List 3 of the elements.

2. Standard Set Proofs

(a) Prove that $A \cap B \subseteq A \cup B$ for any sets A, B.

(b) Prove that $A \cap (A \cup B) = A$ for any sets A, B.

(c) Prove that $A \cap (A \cup B) = A \cup (A \cap B)$ for any sets A, B.

3. Cartesian Product Proof

Write an English proof to show that $A \times C \subseteq (A \cup B) \times (C \cup D)$.

4. Powerset Proof

Suppose that $A \subseteq B$. Prove that $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

5. Set Prove or Disprove

(a) Prove or disprove: For any sets A and B, $A \cup B \subseteq A \cap B$.

(b) Prove or disprove: For any sets A, B, and C, if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

Number Theory

6. Modular Computation

(a) Circle the statements below that are true.
Recall for $a, b \in \mathbb{Z}$: $a|b$ iff $\exists k \in \mathbb{Z}\ (b = ka)$.

(a) $1|3$

(b) $3|1$

(c) $2|2018$

(d) $-2|12$

(e) $1 \cdot 2 \cdot 3 \cdot 4 | 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5$

(b) Circle the statements below that are true.
Recall for $a, b, m \in \mathbb{Z}$ and $m > 0$: $a \equiv b \pmod{m}$ iff $m|(a - b)$.

(a) $-3 \equiv 3 \pmod{3}$

(b) $0 \equiv 9000 \pmod{9}$

(c) $44 \equiv 13 \pmod{7}$

(d) $-58 \equiv 707 \pmod{5}$

(e) $58 \equiv 707 \pmod{5}$

7. Modular Addition

Let m be a positive integer. Prove that if $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then $a + c \equiv b + d \pmod{m}$.

8. Divisibility Proof

Let the domain of discourse be integers. Consider the following claim:

$$\forall n \forall d \ ((d \mid n) \rightarrow (-d \mid n))$$

(a) Translate the claim into English.

(b) Write an English proof that the claim holds.

9. Modular Multiplication

Write an English proof to prove that for an integer $m > 0$ and any integers a, b, c, d, if $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then $ac \equiv bd \pmod{m}$.

10. Another Divisibility Proof

Write an English proof to prove that if k is an odd integer, then $4 \mid k^2 - 1$.