CSE 390Z: Mathematics for Computation Workshop

Week 3 Workshop

Conceptual Review

(a) Translate "all cats are friends with a dog" to predicate logic. Domain of dicourse: mammals.

(b) Inference Rules:

Introduce $\vee$: $\dfrac{A}{\therefore\ A \vee B,\ B \vee A}$ Eliminate $\vee$: $\dfrac{A \vee B\ ;\ \neg A}{\therefore\ B}$

Introduce $\wedge$: $\dfrac{A\ ;\ B}{\therefore\ A \wedge B}$ Eliminate $\wedge$: $\dfrac{A \wedge B}{\therefore\ A\ ,\ B}$

Direct Proof: $\dfrac{A \Rightarrow B}{\therefore\ A \to B}$ Modus Ponens: $\dfrac{A\ ;\ A \to B}{\therefore\ B}$

Intro $\exists$: $\dfrac{P(c)\ \text{for some } c}{\therefore\ \exists x P(x)}$ Eliminate $\exists$: $\dfrac{\exists x P(x)}{\therefore\ P(c)\ \text{for some special } c}$

Intro $\forall$: $\dfrac{P(a);\ a\ \text{is arbitrary}}{\therefore\ \forall x P(x)}$ Eliminate $\forall$: $\dfrac{\forall x P(x)}{\therefore\ P(a);\ \text{for any } a}$

(c) What are DeMorgan's Laws for Quantifiers?

(d)   Given $A \wedge B$, prove $A \vee B$ Given $P \to R,\ R \to S$, prove $P \to S$.

(e) How do you prove a "for all" statement? E.g. prove $\forall x P(x)$. How do you prove a "there exists" statement? E.g. prove $\exists x P(x)$.

1. Trickier Translation

Express the following sentences in predicate logic. The domain of discourse is movies and actors. You may use the following predicates: Movie(x) ::= x is a movie, Actor(x) ::= x is an actor, Features(x, y) ::= x features y.

(a) Every movie features an actor.

(b) Not every actor has been featured in a movie.

(c) All movies that feature Harry Potter must feature Voldermort.
   Hint: You can use "Harry Potter" and "Voldemort" as constants that you can directly plug into a predicate.

(d) There is a movie that features **exactly one** actor.

2. Negating Quantifiers

In the previous question, we translated the sentence "Not every actor has been featured in a movie" to predicate logic.

This was Kriti's translation: $\neg \forall x(\text{Actor}(x) \rightarrow \exists y(\text{Movie}(y) \land \text{Features}(y, x)))$

This was Tanush's translation: $\exists x(\text{Actor}(x) \land \forall y(\text{Movie}(y) \rightarrow \neg \text{Features}(y, x)))$

(a) Azita claims that Kriti and Tanush are both correct. Do you agree with Azita?

(b) Use a chain of predicate logic equivalences to prove that the two translations are equivalent.
   Hint: You may wish to use DeMorgan's Law for Predicates and the Law of Implication.

3. More Tricky Translations

Translate the following English sentences to predicate logic. The domain is integers, and you may use $=$, $\neq$, and $>$ as predicates. Assume the predicates Prime, Composite, and Even have been defined appropriately.
Note: Composite numbers are ones that have at least 2 factors (the opposite of prime).

(a) 2 is prime.

(b) Every **positive** integer is prime or composite, but not both.

(c) There is **exactly one** even prime.

(d) 2 is the only even prime.

(e) Some, but not all, composite integers are even.

4. Propositional Proofs

(a) Prove that given $p \rightarrow q$, we can conclude $(p \wedge r) \rightarrow q$

(b) Prove that given $p \lor q$, $q \to r$, and $r \to s$, we can conclude $\neg p \to s$.

5. Predicate Proofs 1

(a) Prove that $\forall x P(x) \to \exists x P(x)$. You may assume that the domain is nonempty.
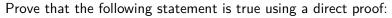
(b) Given $\forall x (T(x) \to M(x))$ and $\exists x (T(x))$, prove that $\exists x (M(x))$.

(c) Given $\forall x(P(x) \rightarrow Q(x))$, prove that $\exists x P(x) \rightarrow \exists y Q(y)$. You may assume that the domain is non-empty.

6. Predicate Proofs 2

Given $\forall x \ (P(x) \lor Q(x))$ and $\forall y \ (\neg Q(y) \lor R(y))$, prove $\exists x \ (P(x) \lor R(x))$. You may assume that the domain is not empty.

7. Direct Proof 1

Prove that the following statement is true using a direct proof:

For all integers n and m, if n and m are odd, then $n + m$ is even.

8. Direct Proof 2

Prove that the following statement is true using a direct proof:

For all integers n, if n is even, then $\frac{n}{2} * n$ is even.

9. Challenge: Predicate Negation

Translate "You can fool all of the people some of the time, and you can fool some of the people all of the time, but you can't fool all of the people all of the time" into predicate logic. Then, negate your translation. Then, translate the negation back into English.

Hint: Let the domain of discourse be all people and all times, and let $P(x, y)$ be the statement "You can fool person x at time y". You can get away with not defining any other predicates if you use P.