CSE 390Z: Mathematics for Computation Workshop

Practice 311 Final

Name: ______________________________

UW ID: ______________________________

Instructions:

- This is a **simulated practice final**.You will be graded on your effort, not correctness, on this exam.
- Nevertheless, please treat this as if it is a real exam. That means that you should not discuss with your neighbors or use your devices during the next hour.
- If you find yourself needing to look concepts up in your notes or lecture materials, feel free to do so. Consider taking note of this so you can include it on your note sheet for the real exam, where you will not be able to have unlimited access to your notes.
- If you get stuck on a problem, consider moving on and coming back later. In the actual exam, there will likely be opportunity for partial credit.
- There are 8 problems on this exam, totaling 100 points.

Question	**Max points**
Translations	12
Language Representations	15
Induction I	15
Induction II	20
Modular Arithmetic	10
Relations	7
Irregularity	20
Grading Morale	1
Total	**100**

1. Translations [12 points]

For this problem, let the domain of discourse be all days of the year 2023 and all people. For each part, translate the English sentence to predicate logic. You may use the following predicates:
Person(x) := x is a person
Birthday(x,y) := x is y's birthday
Holiday(x) := x is a holiday
Weekend(x) := x is on the weekend

(a) [3 points] Everyone has a birthday.

(b) [3 points] No one can have more than one birthday. (Note that a person could have zero birthdays.)

(c) [3 points] Every holiday is also someone's birthday. (Note that the "someone" can be a different person for each holiday).

(d) [3 points] Not all holidays are on the weekend.

2. Language Representations [15 points]

Let the alphabet be $\Sigma = \{a, b\}$.
Consider the language $L = \{w \in \Sigma^* : w$ where every occurence of a is not followed by $bb\}$.

Some strings in L include ϵ, a, ab, $aabaaab$, bbb, and $bbbbababa$.
Some strings **not** in L include abb, $bababb$, $aaabbaa$

(a) (5 points) Give a regular expression that represents L.

(b) (5 points) Give a CFG that represents L.

(c) (5 points) Give a DFA that represents L.

3. Induction I [15 points]

Consider the following recursive definition of a_n:

$$a_1 = 1$$
$$a_2 = 1$$
$$a_n = \frac{1}{2}(a_{n-1} + \frac{2}{a_{n-2}}) \qquad \text{for } n > 2$$

Prove that $1 \leq a_n \leq 2$ for all integers $n \geq 1$.

Hint: To prove $1 \leq x \leq 2$ for some x, you should prove two facts: $1 \leq x$ and $x \leq 2$.

4. Induction II [20 points]

Recall the recursive definition of a list of integers:

- $[\ ]$ is the empty list
- If L is a list and a is an integer, then $a :: L$ is a list whose first element is a, followed by the elements of L.

Consider the following functions defined on lists:
len($[\ ]$) $= 0$
len($x :: L$) $= 1+$ len(L)

inc($[\ ]$) $= [\ ]$
inc($x :: L$) $= (x + 1) ::$ inc(L)

sum($[\ ]$) $= 0$
sum($x :: L$) $= x +$ sum(L)

Prove that for all lists L, sum(inc(L)) $=$ sum(L) $+$ len(L).

5. Modular Arithmetic [10 points]

(a) Prove or disprove: If $a \equiv b \pmod{10}$, then $a \equiv b \pmod{5}$. [5 points]

(b) Prove or disprove: If $a \equiv b \pmod{10}$, then $a \equiv b \pmod{20}$. [5 points]

6. Relations [7 points]

We define the relation R as $\{(x, y) : 3|(x - y)\}$.
List 3 elements of R

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List the properties that R has out of the following: reflexive, transitive, symmetric, antisymmetric: (You do not need to show work for this part, just list the properties.)

7. Irregularity [20 points]

Prove that the set of strings $\{0^n10^n : n \geq 0\}$ is not regular.

8. Just for fun [1 point]

Draw a portrait of yourself doing your favorite winter activity :)