0. Relations

(a) Consider the relation \( R \subseteq \mathbb{Z} \times \mathbb{Z} \) defined by \((a, b) \in R\) iff \(a < b\). Determine if \( R \) is reflexive, symmetric, antisymmetric, and/or transitive. If the relation has a property, explain why. If not, state a counterexample.

**Solution:**
- Reflexive: No. For example, \((0, 0) \notin R\).
- Symmetric: No. For example, \((0, 1) \in R\) but \((1, 0) \notin R\).
- Antisymmetric: Yes. Suppose \((a, b) \in R\) and \(a \neq b\). Then \(a < b\). Then by properties of less than, it is not possible for \(b < a\). So \((b, a) \notin R\).
- Transitive: Yes. Suppose \((a, b) \in R\) and \((b, c) \in R\). Then \(a < b\) and \(b < c\). So \(a < c\). So \((a, c) \in R\).

(b) Given an example of a relation that is neither symmetric nor antisymmetric.

**Solution:**
Consider the relation \( R = \{(0, 1), (1, 0), (1, 2)\} \). This is not symmetric, because \((1, 2) \in R\) but \((2, 1) \notin R\). This is also not antisymmetric, because \((0, 1) \in R\) and \((1, 0) \in R\).

1. Video Solution

Watch [this solution video](#) after making an initial attempt. Then, answer the following questions.

(a) What is one thing you took away from the video solution?

(b) What topic from the quick check or lecture would you most like to review in workshop?