0. How Many Ones?
The set \( T \) is defined as follows:

- **Base case**: \( \epsilon \in T \)
- **Recursive Rules**:
  - If \( x \in T \), then \( 11x \in T \)
  - If \( x \in T \) and \( y \in T \), then \( x0y \in T \)

Given the following recursively defined function

- \( \text{numOnes}(\epsilon) = 0 \)
- \( \text{numOnes}(11x) = 2 + \text{numOnes}(x) \)
- \( \text{numOnes}(x0y) = \text{numOnes}(x) + \text{numOnes}(y) \)

Prove that for all strings \( n \) in \( T \), \( \text{numOnes}(n) \) is even

**Hint**: In structural induction, the structure of your induction mirrors the recursive definition.

**Solution**:

Let \( P(n) \) be "2 \( | \) numOnes(n)". We will show that \( P(n) \) is true for all \( n \in T \) by structural induction.

**Base Case** \( (n = \epsilon) \):

\( \text{numOnes}(\epsilon) = 0 \) definition of \text{numOnes}

\( 0 = 2 \cdot 0 \) and \( 2 \mid 0 \) by definition of divides.

Therefore \( P(\epsilon) \) holds true.

Let \( s \) be an arbitrary element of \( T \) not covered by the base cases. By the exclusion rule, \( s = 11x \) or \( s = x0y \) for some elements \( x, y \in T \).

**Induction Hypothesis**: Suppose \( P(x) \) and \( P(y) \) are true.

**Induction Step**: 

**Case 1**: \( 11x \)

\( \text{numOnes}(11x) = 2 + \text{numOnes}(x) \) by definition of \text{numOnes}. By the inductive hypothesis, 2 \( | \) \text{numOnes}(x).

Therefore, by definition of divides \( \text{numOnes}(x) = 2z \) for some integer \( z \). Thus,

\[
\text{numOnes}(11x) = 2 + \text{numOnes}(x) = 2z + 2 = 2(z + 1)
\]

Therefore, by definition of divides, 2 \( | \) \text{numOnes}(11x). Therefore, \( P(11x) \) holds.

**Case 2**: \( x0y \)
numOnes(x0y) = numOnes(x) + numOnes(y) by definition of numOnes. By the induction hypothesis, 2 | numOnes(x) and 2 | numOnes(y). Therefore, by definition of divides, numOnes(x) = 2z for some integer z and numOnes(y) = 2q for some integer q. Thus,

numOnes(x0y) = numOnes(x) + numOnes(y) = 2z + 2q = 2(z + q)

Therefore, by definition of divides 2 | numOnes(x0y). Therefore, P(x0y) holds.

The result follows for all $n \in T$ by structural induction.

1. **Video Solution**

Watch [this video](#) on the solution after making an initial attempt. Then, answer the following questions.

(a) What is one thing you took away from the video solution?

(b) What topic from the quick check or lecture would you most like to review in workshop?