CSE 390Z: Mathematics for Computation Workshop

Additional Practice Final

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Instructions:

- This is a **simulated practice final** prepared by the CSE 390Z teaching staff for Autumn 2023.
- We make no guarantees regarding the similarity of this material to any real 311 exams, nor do we guarantee that our solutions would earn full points if marked by 311 graders.

Advice:

- You are encouraged to take this exam in an environment that mimics the test conditions of your 311 exam (i.e., limit yourself to one-hour and fifty-minutes, don't use any outside resources besides one piece of 8.5x11 inch paper with handwritten notes, and avoid electronic devices such as calculators).
- Some of the questions on this exam are significantly more difficult than the ones we've encountered in workshop. We want you to feel challenged. Try to not stress if you run out of time or get stuck.
- Remember to take deep breaths.
- Relax, you are here to learn.

Question	Max Points
Training Wheels	13
Good Ol' Proofs	14
Strong Induction	20
Structural Induction	20
Languages	20
Irregularity	10
The Other Stuff	12
Grading Morale	1
Total	**110**

1. Training Wheels [13 points]

For this problem, our domain of discourse is college football teams and college football conferences.

You are allowed to use the $\neq$ symbol to check that two objects are not equivalent.

We will use the following predicates:

- $\texttt{Team}(x) :=$ x is a football team.
- $\texttt{UW}(x) :=$ x is the University of Washington football team.
- $\texttt{WSU}(x) :=$ x is the Washington State University football team.
- $\texttt{OSU}(x) :=$ x is the Oregon State University football team.

- $\texttt{OldPac}(x) :=$ x is the old Pac-12 Conference.
- $\texttt{NewPac}(x) :=$ x is the new Pac-2 Conference.

- $\texttt{Member}(x, y) :=$ the football team x has been a part of the conference y.
- $\texttt{Lost}(x, y) :=$ x lost to y in a football game.

(a) State whether the two statements below are equivalent. Provide a one sentence justification. **[2 points]**

$$\exists y\Big[\texttt{OldPac}(y) \land \forall x\Big(\texttt{Team}(x) \rightarrow \big(\texttt{UW}(x) \rightarrow \texttt{Member}(x, y)\big)\Big)\Big]$$

$$\exists y\Big[\texttt{OldPac}(y) \land \forall x\Big(\texttt{UW}(x) \rightarrow \texttt{Member}(x, y)\Big)\Big]$$

(b) Translate the following sentence into predicate logic. **[3 points]**

Excluding WSU, at least one team has been a part of the new Pac-2 conference and the old Pac-12 conference.

(c) Translate the following statement into predicate logic. **[4 points]**

UW has won against all football teams besides itself, and WSU has lost to all fooootball teams besides itself.

(d) Negate the following statement. Your final answer should have zero negations. **[4 points]**

Warning: this statement makes absolutely no sense. Do **NOT** spend time thinking about its meaning. We want you to blindly follow your equivalency laws here.

$$\forall x \forall y\Big[\big(\texttt{WSU}(x) \land \texttt{OSU}(y)\big) \land \big(\neg\texttt{Lost}(x, y) \lor \neg\texttt{Lost}(y, x)\big)\Big]$$

This page intentionally left blank to provide additional space for your answers to Question 1.

2. Good Ol' Proofs [14 points]

(a) Prove for some predefined sets A, B, C that $(A \setminus B) \cup (C \setminus B) = (A \cup C) \setminus B$. **[8 points]**

Hint: You will need to use proof by cases.

(b) Prove true or false: if $a \equiv 1 \pmod 5$ and $b \equiv 1 \pmod 5$, then $\gcd(a, b) \equiv 1 \pmod 5$. **[3 points]**

(c) Consider the following statement: if $a, b \in \mathbb{Z}$ and $a \geq 2$, then $a \nmid b$ or $a \nmid b + 1$.

Your goal is to disprove this statement using proof by contradiction.

Write out **JUST THE FIRST SENTENCE** of the proof.

Your answer should look something like "Suppose, for the sake of contradiction, ..." **[3 points]**

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3. Strong Induction [20 points]

Consider the function f, which takes a natural number as input and outputs a natural number.

$$f(n) = \begin{cases} 1 & \text{if } n = 0 \\ 2 & \text{if } n = 1 \\ f(n-1) + 2 \cdot f(n-2) & \text{if } n \geq 2 \end{cases}$$

Prove that $f(n) = 2^n$ for all $n \in \mathbb{N}$.

4. Structural Induction [20 points]

We define **List** as such:

- Basis step: $\texttt{nil} \in$ **List**
- Recursive step: if $L \in$ **List** and $a \in \mathbb{Z}$, then $a :: L \in$ **List**

We define the function `len` as such:

- $\texttt{len}(\texttt{nil}) \coloneqq 0$
- $\texttt{len}(a :: L) \coloneqq \texttt{len}(L) + 1$ for any $L \in$ **List** and $a \in \mathbb{Z}$

We define the function `append` as such:

- $\texttt{append}(\texttt{nil}, L) \coloneqq L$ for any $L \in$ **List**
- $\texttt{append}(a :: L, R) \coloneqq a :: \texttt{append}(L, R)$ for any $L, R \in$ **List** and $a \in \mathbb{Z}$

Prove that $\texttt{append}(L, \texttt{append}(R, M)) = \texttt{append}(\texttt{append}(L, R), M)$ for all $L, R, M \in$ **List**. In other words, we are asking you to prove the associative property for `append`.

5. Languages [20 points]

(a) Describe in English the language matched by the regular expression $(0 \cup 1)^*10101(0 \cup 1)^*$. **[2 points]**

(b) Write a regular expression that recognizes all binary strings which do **NOT** contain the substring 110. **[6 points]**

(c) Design a DFA that recognizes all binary strings which end with a 1 and do **NOT** contain the substring 00. **[6 points]**

(d) Design a CFG for the language consisting of all binary non-palindromes. **[6 points]**

6. Irregularity [10 points]

Show that the language $L = \{w \in \{0, 1\} : w$ contains exactly two more 1s than it does 0s$\}$ is irregular.

7. The Other Stuff [12 points]

(a) Let Q be the relation $\{(1,1),(1,3),(1,4),(2,2),(3,1),(3,4),(4,1),(4,3)\}$ on the set $A = \{1,2,3,4\}$.

Consider the following properties: reflexivity, symmetry, anti-symmetry, transitivity. If the property holds, simply state so. If the property does not hold, provide a counterexample. **[2 points]**

(b) Suppose that R and S are symmetric relations on some non-empty set.

Prove or disprove the claim that $R - S$ is symmetric. **[6 points]**

(c) Consider the following claim: "for any set of strings, if it cannot be matched by a regular expression, then we cannot write a Java program that recognizes it."

Is the claim valid? If it's true, write a brief explanation (2-3 sentences) as to why. If it's false, provide a counterexample, explain why your counterexample is irregular, and explain how to design a Java program that recognizes it. **[4 points]**

8. Grading Morale [1 point]

What's one of your go-to jokes?

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