CSE 390Z: Mathematics for Computation Workshop

Week 1 Workshop Problems Solutions

Conceptual Review

(a) What is the difference between an atomic and compound proposition?

Solution:

An atomic proposition cannot be broken into parts that have their own truth values. "I like to run and I like to draw" can be broken down into two statements that have a truth value; "I like to run" and "I like to draw". However, there is no way to break the statement "I like to draw" into a smaller part that has its own truth value.

(b) What does it mean for two propositions to be *equivalent* $(p \equiv q)$?

Solution:

Equivalence is an assertion over all possible truth values that p and q always have the same truth values.

(c) What does it mean for two propositions to be *biconditional* $(p \leftrightarrow q)$?

Solution:

$p \leftrightarrow q$ is a proposition that may be true or false depending on the truth values of the variables p and q. $p \equiv q$ and $p \leftrightarrow q \equiv T$ have the same meaning.

(d) What are two different methods to show that two propositions are equivalent?

Solution:

The first method is to write a truth table for each proposition, and check that each row has the same truth value. The second is to use a chain of equivalences that starts at one proposition and ends at the other.

1. Translation: Running from my problems

Define a set of three atomic propositions, and use them to translate the following sentences.

(i) I am going for a run and it is snowing, or it is not snowing.

(ii) If it's snowing and it's Friday, I am not going for a run.

(iii) I am going for a run only if it is not Friday.

Solution:

p: I am going for a run
q: It is snowing
r: It is Friday

(i) $(p \wedge q) \vee \neg q$

(ii) $(q \wedge r) \rightarrow \neg p$

(iii) $p \rightarrow \neg r$

2. Translation: Age is just a number

Define a set of two atomic propositions, and use them to translate the following sentences.

(i) If Kai is older than thirty, then Kai is older than twenty.

(ii) Kai is older than thirty only if Kai is older than twenty.

(iii) Whenever Kai is older than thirty, Kai is older than twenty.

(iv) Kai being older than twenty is necessary for Kai to be older than thirty.

Solution:

p: Kai is older than thirty
q: Kai is older than twenty

(i) $p \rightarrow q$

(ii) $p \rightarrow q$

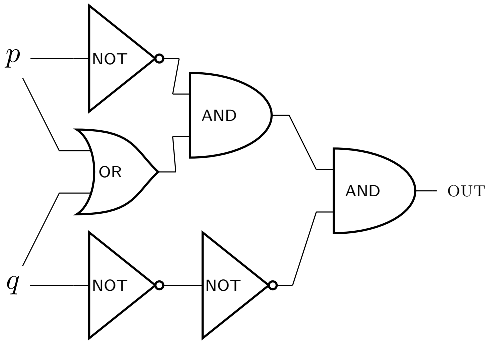
(iii) $p \rightarrow q$

(iv) $p \rightarrow q$

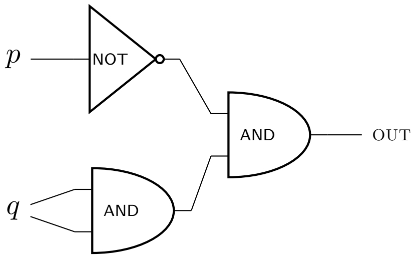
3. Circuits

Convert the following ciruits into logical expressions.

(i)



(ii)



Solution:

(i) $((\neg p) \land (p \lor q)) \land \neg\neg q$

(ii) $\neg p \land (q \land q)$

4. Truth Table

Draw a truth table for $(p \rightarrow \neg q) \rightarrow (r \oplus q)$

Solution:

p	q	r	$\neg q$	$p \rightarrow \neg q$	$r \oplus q$	$(p \rightarrow \neg q) \rightarrow (r \oplus q)$
T	T	T	F	F	F	T
T	T	F	F	F	T	T
T	F	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	T	F	F	T	T	T
F	F	T	T	T	T	T
F	F	F	T	T	F	F

5. Proving Equivalences

We wish to prove that the following is true for all values of the propositions p and q (i.e. that it is a tautology):

$$\neg p \vee ((q \wedge p) \vee (\neg q \wedge p))$$

The following chain of equivalences does this, but it is missing citations for which rules are used. Fill in the blanks with names of the logic equivalences used at each step.

Hint: Reference the Logical Equivalences sheet under the Resources tab on the CSE 311 Course Website.

$$\begin{aligned}
\neg p \vee ((q \wedge p) \vee (\neg q \wedge p)) &\equiv \neg p \vee ((p \wedge q) \vee (\neg q \wedge p)) && \underline{\hspace{4cm}} \\
&\equiv \neg p \vee ((p \wedge q) \vee (p \wedge \neg q)) && \underline{\hspace{4cm}} \\
&\equiv \neg p \vee (p \wedge (q \vee \neg q)) && \underline{\hspace{4cm}} \\
&\equiv \neg p \vee (p \wedge T) && \underline{\hspace{4cm}} \\
&\equiv \neg p \vee p && \underline{\hspace{4cm}} \\
&\equiv p \vee \neg p && \underline{\hspace{4cm}} \\
&\equiv T && \underline{\hspace{4cm}}
\end{aligned}$$

Solution:

$$\begin{aligned}
\neg p \vee ((q \wedge p) \vee (\neg q \wedge p)) &\equiv \neg p \vee ((p \wedge q) \vee (\neg q \wedge p)) && \text{Commutativity} \\
&\equiv \neg p \vee ((p \wedge q) \vee (p \wedge \neg q)) && \text{Commutativity} \\
&\equiv \neg p \vee (p \wedge (q \vee \neg q)) && \text{Distributivity} \\
&\equiv \neg p \vee (p \wedge T) && \text{Negation} \\
&\equiv \neg p \vee p && \text{Identity} \\
&\equiv p \vee \neg p && \text{Commutativity} \\
&\equiv T && \text{Negation}
\end{aligned}$$

6. To Be or Not to Be Equivalent

For each of the following pairs of propositions, determine if the two propositions are equivalent. If they are, prove it using a chain of logical equivalences. If they are not, find an assignment of p and q on which their truth value differ.

(a) $p \wedge q$ vs. $p \vee q$

Solution:

These are not equivalent. For instance, when $p := T, q := F$, the proposition $p \wedge q$ evaluates to F but $p \vee q$ evaluates to T.

(b) $p \rightarrow q$ vs. $\neg q \rightarrow \neg p$
Note: You may not use the Law of Contrapositive in your justification!

Solution:

These are equivalent. Below is the chain of equivalences.

$$\begin{aligned} p \rightarrow q &\equiv \neg p \vee q && \text{Law of Implication} \\ &\equiv \neg p \vee \neg\neg q && \text{Double Negation} \\ &\equiv \neg\neg q \vee \neg p && \text{Commutativity} \\ &\equiv \neg q \rightarrow \neg p && \text{Law of Implication} \end{aligned}$$

(c) $p \rightarrow q$ vs. $q \rightarrow p$

Solution:

These are not equivalent. For instance, when $p := T, q := F$, the proposition $p \rightarrow q$ evaluates to F but $q \rightarrow p$ evaluates to T.

(d) $p \rightarrow q$ vs. $\neg(p \wedge \neg q)$

Solution:

These are equivalent. Below is the chain of equivalences.

$$\begin{aligned} p \rightarrow q &\equiv \neg p \vee q && \text{Law of Implication} \\ &\equiv \neg p \vee \neg\neg q && \text{Double Negation} \\ &\equiv \neg(p \wedge \neg q) && \text{DeMorgan's Law} \end{aligned}$$

7. Conditional Logic

You've already seen how implications translate to English and back, but we can also translate them to code. In English, $p \rightarrow q$ translates to "If p, then q". We might also translate this (in Java) to a conditional:

```
if (p) {
    q
}
```

Lets say we have propositions p, q, and r. It may be helpful in the following problem to think of p and q as booleans, and r as a print statement.

(a) Convert each of the following propositions to Java (or pseudo-) code:

$p \to (q \to r)$

Solution:

```
if (p) {
    if (q) {
        r
    }
}
```

$(p \wedge q) \to r$

Solution:

```
if (p && q) {
    r
}
```

(b) Based on this, do you think $p \to (q \to r)$ and $(p \wedge q) \to r$ are logically equivalent? Why or why not?

Solution:

Left as an exercise due to similarity to a homework problem.

8. Propositions in the wild

Give a real-life example of statements p, q, and r such that p and q together imply r, but neither p nor q alone imply r.

Solution:

There is no single correct answer for this, but here is an example:
p : I'm older than 12
q : I'm younger than 20
r : I'm a teen