CSE 390Z: Mathematics for Computation Workshop

Practice 311 Final

Name: ________________________________

UW ID: ______________________________

Instructions:

• This is a simulated practice final. You will not be graded on your performance on this exam.

• This final was written to take 50 minutes. The real final will be an hour and 50 minutes.

• Nevertheless, please treat this as if it is a real exam. That means that you may not discuss with your neighbors, reference outside material, or use your devices during the next 50 minute period.

• If you get stuck on a problem, consider moving on and coming back later. In the actual exam, there will likely be opportunity for partial credit.

• There are 5 problems on this exam.
1. All the Machines! [15 points]
Let the alphabet be $\Sigma = \{a, b\}$. Consider the language $L = \{w \in \Sigma^* : \text{every } a \text{ has a } b \text{ two characters later}\}$.
In other words, $L$ is the language of all strings in the alphabet $a, b$ where after any $a$, the character after the $a$ can be anything, but the character after that one must be a $b$.

Some strings in $L$ include $\varepsilon, abb, aabb, bbbabb$. Some strings not in $L$ include $a, ab, aab, ababb$. Notice that the last two characters of the string cannot be an $a$.

(a) (5 points) Give a regular expression that represents $L$.

(b) (5 points) Give a CFG that represents $L$.

(c) (5 points) Give a DFA that represents $L$. 
2. Induction 1 [20 points]
Recall the recursive definition of a list of integers:

- \([\ ]\) is the empty list
- If \(L\) is a list and \(a\) is an integer, then \(a :: L\) is a list whose first element is \(a\), followed by the elements of \(L\).

Consider the following functions defined on lists:
len([ ]) = 0
len(x :: L) = 1 + len(L)

inc([ ]) = []
inc(x :: L) = (x + 1) :: inc(L)

sum([ ]) = 0
sum(x :: L) = x + sum(L)

Prove that for all lists \(L\), sum(inc(L)) = sum(L) + len(L).
3. Induction 2 [20 points]
Consider the following recursive definition of $a_n$:

\[ a_1 = 1 \]
\[ a_2 = 1 \]
\[ a_n = \frac{1}{2} (a_{n-1} + \frac{2}{a_{n-2}}) \quad \text{for } n > 2 \]

Prove that $1 \leq a_n \leq 2$ for all integers $n \geq 1$. 
4. Modular Arithmetic [10 points]
(a) Prove or disprove: If \( a \equiv b \) (mod 10), then \( a \equiv b \) (mod 5). [5 points]

(b) Prove or disprove: If \( a \equiv b \) (mod 10), then \( a \equiv b \) (mod 20). [5 points]
5. **Irregularity** [20 points]
Prove that the set of strings \( \{0^n1^n : n \geq 0\} \) is not regular.