CSE 390Z: Mathematics of Computing

Week 8 Workshop

Conceptual Review

Space to take notes on Structural Induction, Regular Expressions, and CFGs:

1. Structural Induction x CFG Example

Consider the following CFG:

$$S \to SS \mid 0S1 \mid 1S0 \mid \epsilon$$

Prove that every string generated by this CFG has an equal number of 1's and 0's.

Hint: You may wish to define the functions $\#_0(x), \#_1(x)$ on a string x.

2. Context Free Grammars

Consider the following CFG which generates strings from the language V $:= \{0, 1, 2, 3, 4\}^*$

$$\mathbf{S} \rightarrow 0\mathbf{X}4$$
$$\mathbf{X} \rightarrow 1\mathbf{X}3 \mid 2$$

List 5 strings generated by the CFG and 5 strings from V not generated by the CFG. Then, summarize this CFG in your own words.

3. Constructing Languages

For each of the following, construct a regular expression and a CFG for the specified language.

(a) Strings from the language $S := \{a\}^*$ with an even number of a's.

(b) Strings from the language $S := \{a, b\}^*$ with odd length.

(c) (Challenge) Strings from the language $S := \{a, b\}^*$ with an even number of a's or an odd number of b's.

4. Structural Induction on Palindromes

Consider the following *recursive* defintion of the set B of palindrome binary strings:

- **Base case:** $\varepsilon \in B$, $0 \in B$, $1 \in B$.
- **Recursive steps**:
  - If $s \in B$, then $0s0 \in E$, $1s1 \in B$, and $ss \in B$.

  Now define the functions numOnes(x) and numZeros(x) to be the number of 1s and 0s respectively in the string x.

Use *structural induction* to prove that for any string $s \in B$, numOnes(s) $\cdot$ numZeros(s) is even.

5. Relations

Suppose A is nonempty set and $R, S \subset A \times A$. The universe that A exists in is only integers.

(a) Prove or disprove: If R and S are reflexive, $R \cap S$ is reflexive.

(b) Prove or disprove: If R and S are transitive, $R \cup S$ is transitive.

(c) Prove or disprove: If R is symmetric, $\overline{R}$ is symmetric.