CSE 390Z: Mathematics for Computation Workshop

Week 5 Workshop

Name: ______________________________ Collaborators: ______________________________

Conceptual Review

(a) **Set Definitions**

Set Equality: $A = B := \forall x(x \in A \leftrightarrow x \in B)$
Subset: $A \subseteq B := \forall x(x \in A \rightarrow x \in B)$
Union: $A \cup B := \{x \ : \ x \in A \vee x \in B\}$
Intersection: $A \cap B := \{x \ : \ x \in A \wedge x \in B\}$
Set Difference: $A \setminus B = A - B := \{x \ : \ x \in A \wedge x \notin B\}$
Set Complement: $\overline{A} = A^C := \{x \ : \ x \notin A\}$
Powerset: $\mathcal{P}(A) := \{B \ : B \subseteq A\}$
Cartesian Product: $A \times B := \{(a, b) \ : a \in A, \ b \in B\}$

(b) How do we prove that for sets A and B, $A \subseteq B$?

(c) How do we prove that for sets A and B, $A = B$?

(d) What does $\{x \in \mathbb{Z} \ : \ x > 0\}$ mean? **Note:** this notation is called "set-builder" notation.

1. Examples

(a) Prove that $A \cap B \subseteq A \cup B$.

(b) Prove that $A \cap (A \cup B) = A \cup (A \cap B)$ with a chain of equivalences proof.

2. Set Operations

Let $A = \{1, 2, 5, 6, 8\}$ and $B = \{2, 3, 5\}$.

(a) What is the set $A \cap (B \cup \{2, 8\})$?

(b) What is the set $\{10\} \cup (A \setminus B)$?

(c) What is the set $\mathcal{P}(B)$?

(d) How many elements are in the set $A \times B$? List 3 of the elements.

3. Set Equality Proof

(a) Write an English proof to show that $A \cap (A \cup B) \subseteq A$ for any sets A, B.

(b) Write an English proof to show that $A \subseteq A \cap (A \cup B)$ for any sets A, B.

(c) Combine part (a) and (b) to conclude that $A \cap (A \cup B) = A$ for any sets A, B.

(d) Prove $A \cap (A \cup B) = A$ again, but using a **chain of equivalences proof** instead.

4. Subsets

Prove or disprove: for any sets A, B, and C, if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

5. $\cup \to \cap$?

Prove or disprove: for all sets A and B, $A \cup B \subseteq A \cap B$.

6. Cartesian Product Proof

Write an English proof to show that $A \times C \subseteq (A \cup B) \times (C \cup D)$.

7. Set Equality Proof

We want to prove that $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$.

(a) First prove this with a chain of logical equivalences proof.

(b) Now prove this with an English proof that is made of two subset proofs.

8. Constructing Sets

Use set builder notation to construct the following sets. You may use arithmetic predicates $=, <, >, \leq, \geq, \neq$, and arithmetic operations $+, \cdot, -, \div$.

Recall that integers are the numbers $\{... - 2, -1, 0, 1, 2...\}$, and are denote $\mathbb{Z}$.

(a) The set of even integers.

(b) The set of integers that are one more than a perfect square.

(c) The set of integers that are greater than 5.

9. Making a Difference

Garrett and Shaoqi are working on their AI homework and tell you the following. Let G denote the set of AI homework questions that Garrett has not yet solved. Let S denote the set of AI homework questions that Shaoqi has not yet solved. Garrett and Shaoqi claim that $G \setminus S = S \setminus G$.

In what circumstance is this true? In what circumstance is it false? Can you justify this (formal proof not required)?