CSE 390Z: Mathematics for Computation Workshop

Week 3 Workshop

Name: ______________________________

Collaborators: ______________________________

Conceptual Review

(a) What are the inference rules?

(b) Given $A \wedge B$, prove $A \vee B$

Given $P \to R$, $R \to S$, prove $P \to S$.

(c) How do you prove a "for all" statement? E.g. prove $\forall x P(x)$

(d) How do you prove a "there exists" statement? E.g. prove $\exists x P(x)$

1. Tricky Translations

Translate the following English sentences to predicate logic. The domain is integers, and you may use $=$, $\neq$, and $>$ as predicates. Assume the predicates Prime, Composite, and Even have been defined appropriately.

(a) 2 is prime.

(b) Every positive integer is prime or composite, but not both.

(c) There is exactly one even prime.

(d) 2 is the only even prime.

2. Formal Proofs: Modus Ponens

(a) Prove that given $p \to q$, $\neg s \to \neg q$, and p, we can conclude s.

(b) Prove that given $\neg(p \vee q) \to s$, $\neg p$, and $\neg s$, we can conclude q.

3. Formal Proofs: Direct Proof Rule

(a) Prove that given $p \to q$, we can conclude $(p \land r) \to q$

(b) Prove that given $p \lor q$, $q \to r$, and $r \to s$, we can conclude $\neg p \to s$.

4. Predicate Logic Formal Proof

(a) Prove that $\forall x P(x) \to \exists x P(x)$. You may assume that the domain is nonempty.

(b) Given $\forall x(T(x) \to M(x))$ and $\exists x(T(x))$, prove that $\exists x(M(x))$.

(c) Given $\forall x(P(x) \to Q(x))$, prove that $(\exists x P(x)) \to (\exists y Q(y))$.

5. Predicate Logic Formal Proof: Oddly Even

Write a formal proof to show: If n, m are odd, then $n + m$ is even.
Let the predicates $\text{Odd}(x)$ and $\text{Even}(x)$ be defined as follows where the domain of discourse is integers:

$$\text{Odd}(x) := \exists y \; (x = 2y + 1)$$
$$\text{Even}(x) := \exists y \; (x = 2y)$$

6. More Formal Proofs: Predicate Logic!

Given $\forall x \; (P(x) \lor Q(x))$ and $\forall y \; (\neg Q(y) \lor R(y))$, prove $\exists x \; (P(x) \lor R(x))$. You may assume that the domain is not empty.

7. More Predicate Translation!

(a) Translate the following sentence into predicate logic:

"Every element in the array A appears at least twice"

Assume the length of the array is a positive even number n, and assume the domain of discourse is valid integer indexes of A, i.e. $\{0, 1, 2, ..., n-1\}$. You should use array notation to refer to an element of the array (i.e. $A[i]$ is the i-*th* element of the array A), and may use $=$. You should not use any other predicates you create yourself.

(b) Now translate the following modified sentence into predicate logic:

"Every element in the array A appears exactly twice"

Hint: You should be able to use what you wrote for part (a), with some additions.

8. More Formal Proofs: Propositional Logic!

Use a formal proof to show that for any propositions a, b, c, the following holds.

$$a \rightarrow (b \rightarrow (c \rightarrow ((a \wedge b) \wedge c)))$$