# CSE 390Z: Mathematics for Computation Workshop

## Week 2 Workshop Problems

## Conceptual Review

(a) What is DNF form? What is CNF form?

(b) What is a domain of discourse?

(c) How do you restrict to a smaller domain in a "for all"? How do you restrict to a smaller domain in an "exists"?

(d) What is the difference between $\forall x \exists y (P(x, y))$ and $\exists y \forall x (P(x, y))$?

(e) What are DeMorgan's Laws for Quantifiers?

## 1. Equivalences: Propositional Logic

Prove $((p \wedge q) \rightarrow r) \equiv (p \rightarrow r) \vee (q \rightarrow r)$ via equivalences. Note that with propositional logic, you are expected to show all steps, including commutativity and associativity.

## 2. Equivalences: Boolean Algebra

(a) Prove $p' + p \cdot q + q' \cdot p = 1$ via equivalences. Note that with Boolean Algebra, you may skip commutativity associativity in the steps that you show.

(b) Prove $(p' + q) \cdot (q + p) = q$ via equivalences.

## 3. DNFs and CNFs

Consider the following boolean functions $A(p, q, r)$ and $B(p, q, r)$.

| $p$ | $q$ | $r$ | $A(p,q,r)$ | $B(p,q,r)$ |
|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 |

(a) Write the DNF (sum of products) and CNF (product of sums) expressions for $A(p, q, r)$.

(b) Write the DNF (sum of products) and CNF (product of sums) expressions for $B(p, q, r)$.

## 4. Domains of Discourse

For the following, find a domain of discourse where the following statement is true and another where it is false. Note that for the arithmetic symbols to make sense, the domains of discourse should be sets of numbers.

(a) $\exists x(2x = 0)$

(b) $\forall x \exists y(x + y = 0)$

(c) $\exists x \forall y(x + y = y)$

## 5. Predicate Logic Gotchas

Let the domain of discourse be all animals. Let Cat$(x)$ ::= "$x$ is a cat" and Blue$(x)$ ::= "$x$ is blue". Translate the following statements to English.

(a) $\forall x(\text{Cat}(x) \wedge \text{Blue}(x))$

(b) $\forall x(\text{Cat}(x) \rightarrow \text{Blue}(x))$

(c) $\exists x(\text{Cat}(x) \wedge \text{Blue}(x))$

Kabir translated the sentence "there exists a blue cat" to $\exists x(\text{Cat}(x) \rightarrow \text{Blue}(x))$. This is wrong! Let's understand why.

(d) Use the Law of Implications to rewrite Kabir's translation without the $\rightarrow$.

(e) Translate the predicate from (d) back to English. How does this differ from the intended meaning?

(f) This is a warning to be very careful when including an implication nested under an exists! It should almost always be avoided, unless there is a forall involved as well. (Nothing to write for this part).

## 6. English to Predicate Logic

Express the following sentences in predicate logic. The domain of discourse is penguins. You may use the following predicates: Love$(x, y)$ ::= "$x$ loves $y$", Dances$(x)$ ::= "$x$ dances", Sings$(x)$ ::= "$x$ sings".

(a) There is a penguin that every penguin loves.

(b) All penguins that sing love a penguin that does not sing.

(c) There is exactly one penguin that dances.

(d) There exists a penguin that loves itself, but hates (does not love) every other penguin.

## 7. Predicate Logic to English

Translate the following sentences to English. Assume the same predicates and domain of discourse as the previous problem.

(a) $\neg\exists x(\text{Dances}(x))$

(b) $\exists x\forall y(\text{Loves}(x,y))$

(c) $\forall x(\text{Dances}(x) \to \exists y(\text{Loves}(y,x)))$

(d) $\exists x\forall y((\text{Dances}(y) \wedge \text{Sings}(y)) \to \text{Loves}(x,y))$

## 8. Predicate Challenge!!

Translate "You can fool all of the people some of the time, and you can fool some of the people all of the time, but you can't fool all of the people all of the time" into predicate logic. Then, negate your translation. Then, translate the negation back into English.

**Hint:** Let the domain of discourse be all people and all times, and let $P(x,y)$ be the statement "You can fool person $x$ at time $y$". You can get away with not defining any other predicates if you use $P$.