QuickCheck: Relations Solutions

Please submit a response to the following questions on Gradescope. We do not grade on accuracy, so please submit your best attempt. You may either typeset your responses or hand-write them. Note that hand-written solutions must be legible to be graded.

We have created this template if you choose to typeset with Latex. This guide has specific information about scanning and uploading pdf files to Gradescope.

0. Relations
(a) Consider the relation \( R \subseteq \mathbb{Z} \times \mathbb{Z} \) defined by \((a, b) \in R \iff a < b\). Determine if \( R \) is reflexive, symmetric, antisymmetric, and/or transitive. If a relation has a property, explain why. If not, state a counterexample.

Solution:
- Reflexive: No. For example, \((0, 0) \notin R\).
- Symmetric: No. For example, \((0, 1) \in R \) but \((1, 0) \notin R\).
- Antisymmetric: Yes. Suppose \((a, b) \in R \) and \( a \neq b \). Then \( a < b \). Then by properties of less than, it is not possible for \( b < a \). So \((b, a) \notin R\).
- Transitive: Yes. Suppose \((a, b) \in R \) and \((b, c) \in R\). Then \( a < b \) and \( b < c \). So \( a < c \). So \((a, c) \in R\).

(b) Given an example of a relation that is neither symmetric nor antisymmetric.

Solution:
Consider the relation \( R = \{ (0, 1), (1, 0), (1, 2) \} \). This is not symmetric, because \((1, 2) \in R \) but \((2, 1) \notin R \).
This is also not antisymmetric, because \((0, 1) \in R \) and \((1, 0) \in R \).

1. Video Solution
Watch this solution video after making an initial attempt. Then, answer the following questions.

(a) What is one thing you took away from the video solution?

(b) What topic from the quick check or lecture would you most like to review in workshop?