

# CSE 390Z: Mathematics for Computation Workshop

## QuickCheck: Induction Solutions (due Sunday, November 13)

Please submit a response to the following questions on Gradescope. We do not grade on accuracy, so please submit your best attempt. You may either typeset your responses or hand-write them. Note that hand-written solutions must be legible to be graded.

We have created **this template** if you choose to typeset with Latex. **This guide** has specific information about scanning and uploading pdf files to Gradescope.

### 0. Induction

Prove by induction that for all integers  $n \geq 1$ :

$$\sum_{i=1}^n 5i = \frac{5n(n+1)}{2}$$

#### Solution:

1. Let  $P(n)$  be the statement  $\sum_{i=1}^n 5i = \frac{5n(n+1)}{2}$ . We prove  $P(n)$  for all integers  $n \geq 1$  by induction.

2. Base Case: When  $n = 1$ , the left-hand side is  $5 * n = 5 * 1 = 5$ . The right-hand side is  $\frac{5n(n+1)}{2} = \frac{5(1+1)}{2} = \frac{5 * 2}{2} = 5$ . Since  $5 = 5$ , the base case holds.

3. Inductive Hypothesis: Suppose that  $P(k)$  holds for some arbitrary integer  $k \geq 1$ . Then  $\sum_{i=1}^k 5i = \frac{5k(k+1)}{2}$ .

4. Inductive Step: Observe that...

$\sum_{i=1}^{k+1} 5i = \left(\sum_{i=1}^k 5i\right) + 5(k+1)$	Definition of Sum
$= \frac{5k(k+1)}{2} + 5(k+1)$	By IH
$= \frac{5k(k+1)}{2} + \frac{10(k+1)}{2}$	Algebra
$= \frac{5k(k+1) + 10(k+1)}{2}$	Algebra
$= \frac{(5k+10)(k+1)}{2}$	Algebra
$= \frac{5(k+2)(k+1)}{2}$	Algebra
$= \frac{5(k+1)(k+2)}{2}$	Algebra

Thus  $\sum_{i=1}^{k+1} 5i = \frac{5(k+1)(k+2)}{2}$ . So  $P(k+1)$  holds.

5. Thus we have proven  $P(n)$  for all integers  $n \geq 1$  by induction.

## 1. Video Solution

Watch **this** solution video **after** making an initial attempt. Then, answer the following questions.

- (a) What is one thing you took away from the video solution?
- (b) What is one way you plan to (or already have) prepared for the 311 midterm?