

## CSE 390Z: Mathematics for Computation Workshop

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### QuickCheck: Set Theory Proof Solutions (due Sunday, November 6)

Please submit a response to the following questions on Gradescope. We do not grade on accuracy, so please submit your best attempt. You may either typeset your responses or hand-write them. Note that hand-written solutions must be legible to be graded.

We have created [this template](#) if you choose to typeset with Latex. [This guide](#) has specific information about scanning and uploading pdf files to Gradescope.

#### 0. Set Proof: A Complement Makes all the Difference

Prove that for any sets  $A, B$ :

$$A \cap \overline{(A \setminus B)} = A \cap B$$

##### Solution:

Let  $A$  and  $B$  be arbitrary sets. First we show  $A \cap \overline{(A \setminus B)} \subseteq A \cap B$ . Let  $x$  be an arbitrary element of  $A \cap \overline{(A \setminus B)}$ . By definition of  $\cap$  and complement,  $x$  is an element of  $A$  and is not an element of  $(A \setminus B)$ . By definition of set difference this means,  $x \in A \wedge \neg(x \in A \wedge x \notin B)$ . By DeMorgan's law we have:  $x \in A \wedge (x \notin A \vee x \in B)$ . Distributing we find,  $(x \in A \wedge x \notin A) \vee (x \in A \wedge x \in B)$ . By definition of empty set, union, and intersection we find:  $(x \in A \wedge x \notin A) \vee (x \in A \wedge x \in B) = \emptyset \cup (A \cap B) = A \cap B$ .

Therefore, since  $x$  was arbitrary we have found every element in  $A \cap \overline{(A \setminus B)}$  is in  $A \cap B$ , so it follows that  $A \cap \overline{(A \setminus B)} \subseteq A \cap B$ .

Now we show  $A \cap B \subseteq A \cap \overline{(A \setminus B)}$ . Let  $x$  be an arbitrary element of  $A \cap B$ . Then, by definition of intersection, we know  $(x \in A \wedge x \in B)$ . By identity, we can state  $(x \in A \wedge x \in B) \vee (x \in A \wedge x \notin A)$ . By definition of distributivity we have,  $x \in A \wedge (x \in B \vee x \notin A)$ . Then by DeMorgan's law we have  $x \in A \wedge \neg(x \in A \wedge x \notin B)$ . Then by definition of intersection, complement, and set difference we have  $A \cap \overline{(A \setminus B)}$ . Therefore, since  $x$  was arbitrary we have found that every element in  $A \cap B$  is in  $A \cap \overline{(A \setminus B)}$ , thus  $A \cap B \subseteq A \cap \overline{(A \setminus B)}$ .

Since we have shown subset equality in both directions, we have proven  $A \cap \overline{(A \setminus B)} = A \cap B$ .

#### 1. Video Solution

Watch [this solution video](#) from our past instructor Omar **after** making an initial attempt. Then, answer the following questions.

- What is one thing you took away from the video solution?
- What topic from the quick check or lecture would you most like to review in workshop?