QuickCheck: Equivalences Solutions

Please submit a response to the following questions on Gradescope. We do not grade on accuracy, so please submit your best attempt. You may either typeset your responses or hand-write them. Note that hand-written solutions must be legible to be graded.

We have created this template if you choose to typeset with Latex. This guide has specific information about scanning and uploading pdf files to Gradescope.

0. Equivalences
Consider the proposition \((p \rightarrow q) \vee \neg(q \land \neg p)\).
(a) Use a truth table to show that the proposition is a tautology.

<table>
<thead>
<tr>
<th>(p)</th>
<th>(q)</th>
<th>(p \rightarrow q)</th>
<th>(\neg p)</th>
<th>(q \land \neg p)</th>
<th>(\neg(q \land \neg p))</th>
<th>((p \rightarrow q) \vee \neg(q \land \neg p))</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
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<td>F</td>
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</tbody>
</table>

(b) Use a chain of equivalences to show that the proposition is a tautology.

Solution:

\[(p \rightarrow q) \vee \neg(q \land \neg p) \equiv (\neg p \lor q) \lor \neg(q \land \neg p)\] Law of Implication
\[\equiv (\neg p \lor q) \lor (\neg q \lor \neg \neg p)\] DeMorgan’s Law
\[\equiv (\neg p \lor q) \lor (\neg q \lor p)\] Double Negation
\[\equiv ((\neg p \lor q) \lor \neg q) \lor p\] Associativity
\[\equiv (\neg p \lor (q \lor \neg q)) \lor p\] Associativity
\[\equiv (\neg p \lor T) \lor p\] Negation
\[\equiv T \lor p\] Domination
\[\equiv p \lor T\] Commutativity
\[\equiv T\] Domination

1. Video Solution
Watch this video on the solution after making an initial attempt. Then, answer the following questions.

(a) What is one thing you took away from the video solution?

(b) What topic from the quick check or lecture would you most like to review in workshop?