Week 5

CSE 390Z
November 1, 2022

Welcome in!

Grab your nametag and the workshop problems from the front

Announcements

New Late Policy posted on Ed

Can submit up to 2 days late (Tuesday at 11:59pm) on all assignments. No further extensions will be granted.

Previous Assignments

- Quick Check 4 — **Late Due**: Today at 11:59 PM

Upcoming Assignments

- Quick Check 5 — **Due:** Sunday November 6, 11:59 PM
- Homework Corrections 1 — **Due:** Sunday November 6, 11:59 PM

Mid-Quarter Announcements

- **Next workshop will be a full-length simulated midterm**

  Taking this simulated midterm and submitting it to Gradescope is required for CSE 390z course credit.

- **Mid-Quarter Feedback**

  Please fill out the survey at this link regarding your 390z experience thus far.

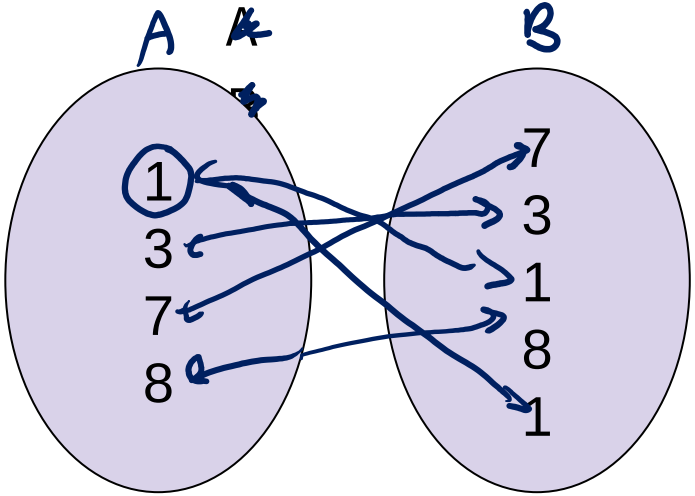
  https://tinyurl.com/390zau22

Conceptual Review

Set Definitions

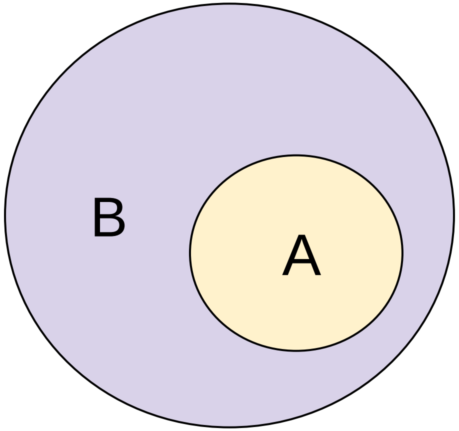
Equality: A = B

$\forall x (x \in A \leftrightarrow x \in B)$



Subset: A ⊆ B

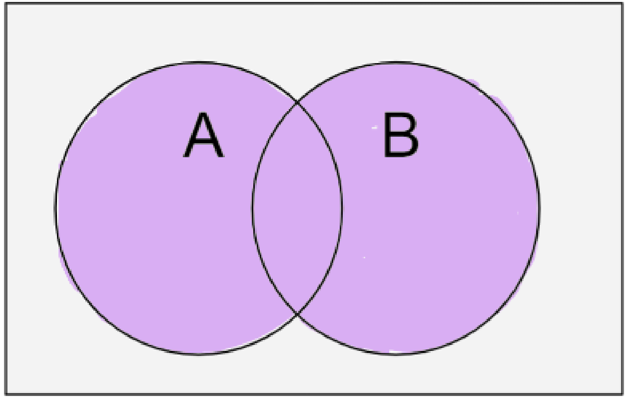
$\forall x (x \in A \rightarrow x \in B)$



Set Definitions

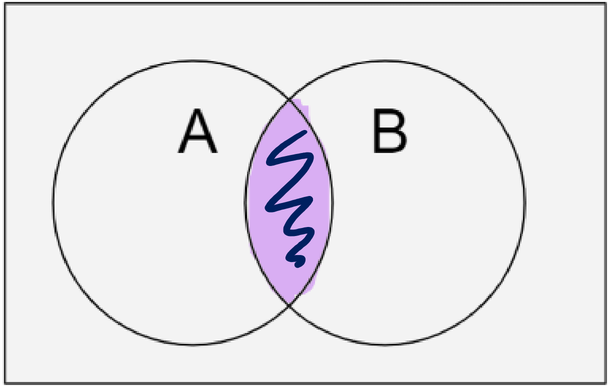
Union: $A \cup B$

$\{x : x \in A \vee x \in B\}$



Intersection: $A \cap B$

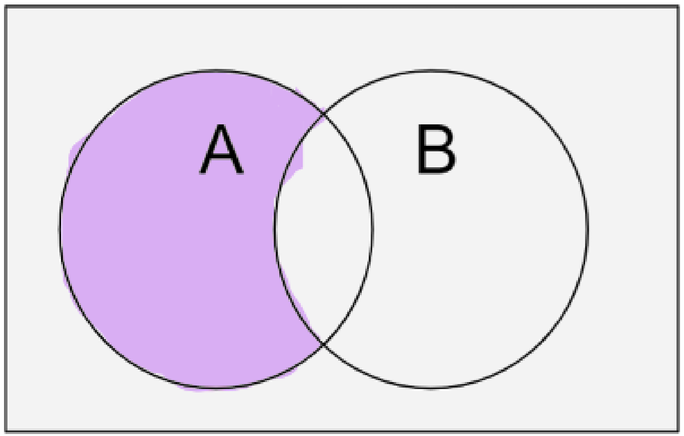
$\{x : x \in A \wedge x \in B\}$



Set Definitions

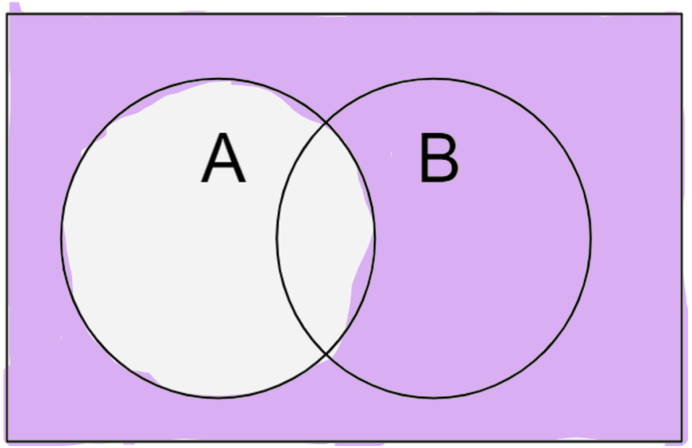
Set Difference: $A \setminus B$

$\{x : x \in A \land x \notin B\}$

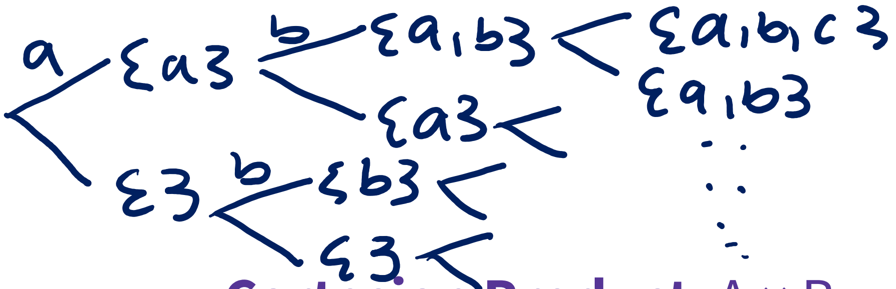


Set Complement: A

$\{x : x \notin A\}$



Set Definitions



Powerset: $\mathcal{P}(A)$

$\{B : B \subseteq A\}$

A has n

P(A) 2^n

A = {a,b,c}

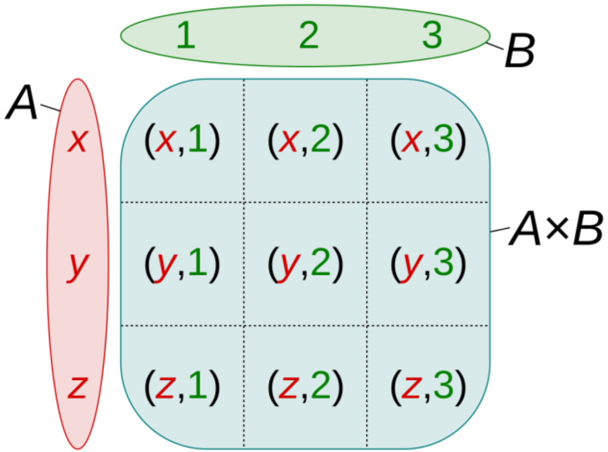
{
{},
{a}, {b}, {c},
{a,b}, {a,c}, {b,c},
{a,b,c}
}

$A \in P(A)$

Cartesian Product: $A \times B$

$\{(a, b) : a \in A, b \in B\}$

(a,b)



Set Proofs

(b) How do we prove that for sets A and B, $A \subseteq B$?

[let $x \in A$ be arbitrary... show $x \in B$. Thus, $A \subseteq B$

(c) How do we prove that for sets A and B, $A = B$?

"Meta Theorem"

① $A \subseteq B$ and $B \subseteq A$

② Chain of Equivalences

$x \in A \equiv$ ___

$\equiv$ ___

$\equiv x \in B$

(d) What does $\{x \in \mathbb{Z} : x > 0\}$ mean?

Set of positive integers

Set Proofs

(b) How do we prove that for sets A and B, $A \subseteq B$?

Let $x \in A$ be arbitrary… thus $x \in B$. Since x was arbitrary, $A \subseteq B$.

(c) How do we prove that for sets A and B, $A = B$?

Method 1: Use two subset proofs to show that $A \subseteq B$ and $B \subseteq A$.

Method 2: Use a chain of logical equivalences.

(d) What does $\{x \in Z : x > 0\}$ mean?

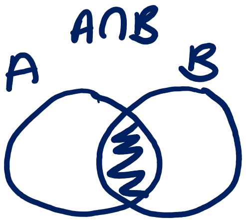
The set of all positive integers.

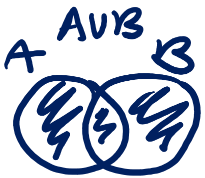
Example Problems

(a) Prove that $A \cap B \subseteq A \cup B$

x

Intuition





English

Let $x \in A \cap B$ be arb. Then $x \in A$ and $x \in B$. So certainly, $x \in A$. Then $x \in A$ or $x \in B$. By def of union, $x \in A \cup B$. Since x arb., $A \cap B \subseteq A \cup B$.

Formal

1. Let x be arbitrary

2.1 $x \in A \cap B$ (Assumption)

2.2 $x \in A \land x \in B$ (Def $\cap$)

2.3 $x \in A$ (Elim $\land$)

2.4 $x \in A \lor x \in B$ (Intro $\lor$)

2.5 $x \in A \cup B$ (Def $\cap$)

2. $x \in A \cap B \to x \in A \cup B$ DPR

3. $\forall x (x \in A \cap B \to x \in A \cup B)$

4. $A \cap B \subseteq A \cup B$ Def $\subseteq$

(a) Prove that $A \cap B \subseteq A \cup B$

Let $x \in A \cap B$ be arbitrary.

Then by definition of intersection, $x \in A$ and $x \in B$.

So certainly $x \in A$ or $x \in B$.

Then by definition of union, $x \in A \cup B$.

Since x was arbitrary, we have shown $A \cap B \subseteq A \cup B$

(b) Prove that $A \cap (A \cup B) = A \cup (A \cap B)$

Strategy ①

Show

$A \cap (A \cup B) \subseteq A \cup (A \cap B)$

AND

show

$A \cup (A \cap B) \subseteq A \cap (A \cup B)$

Strategy ②

Let x be arbitrary.

$x \in A \cap (A \cup B) \equiv x \in A \wedge x \in A \cup B$

$\equiv x \in A \wedge (x \in A \vee x \in B)$

$\equiv (x \in A \wedge x \in A) \vee (x \in A \wedge x \in B)$

$\equiv (x \in A) \vee (x \in A \wedge x \in B)$

$\equiv x \in A \vee (x \in (A \cap B))$

$\equiv x \in (A \cup (A \cap B))$

Thus $A \cap (A \cup B) = A \cup (A \cap B)$

(b) Prove that $A \cap (A \cup B) = A \cup (A \cap B)$

Let x be arbitrary. Observe that:

$x \in A \cap (A \cup B)$	$\equiv (x \in A) \land (x \in A \cup B)$	Def of Intersection
	$\equiv (x \in A) \land ((x \in A) \lor (x \in B))$	Def of Union
	$\equiv ((x \in A) \land (x \in A)) \lor ((x \in A) \land (x \in B))$	Distributivity
	$\equiv (x \in A) \lor ((x \in A) \land (x \in B))$	Idempotency
	$\equiv (x \in A) \lor (x \in A \cap B)$	Def of Intersection
	$\equiv x \in A \cup (A \cap B)$	Def of Union

Since x was arbitrary, we have shown $A \cap (A \cup B) = A \cup (A \cap B)$.

Workshop Problems