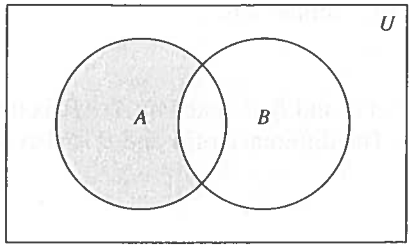
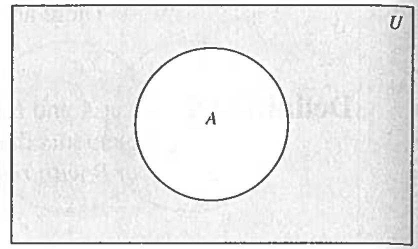
LIST OF SYMBOLS

TOPIC	SYMBOL	MEANING	PAGE
LOGIC	$\neg p$	negation of p	3
	$p \wedge q$	conjunction of p and q	4
	$p \vee q$	disjunction of p and q	4
	$p \oplus q$	exclusive or of p and q	5
	$p \rightarrow q$	implication p implies q	6
	$p \leftrightarrow q$	biconditional of p and q	10
	$p \equiv q$	equivalence of p and q	27
	$\mathbf{T}$	tautology	29
	$\mathbf{F}$	contradiction	29
	$P(x_1, \ldots, x_n)$	propositional function	42
	$\forall x P(x)$	universal quantification of $P(x)$	44
	$\exists x P(x)$	existential quantification of $P(x)$	45
	$\exists! x P(x)$	uniqueness quantification of $P(x)$	46
	$\therefore$	therefore	73
	$p\{S\}q$	partial correctness of S	393
SETS	$x \in S$	x is a member of S	122
	$x \notin S$	x is not a member of S	122
	$\{a_1, \ldots, a_n\}$	list of elements of a set	122
	$\{x \mid P(x)\}$	set builder notation	122
	$\mathbf{N}$	set of natural numbers	122
	$\mathbf{Z}$	set of integers	122
	$\mathbf{Z}^+$	set of positive integers	122
	$\mathbf{Q}$	set of rational numbers	122
	$\mathbf{R}$	set of real numbers	122
	$[a, b]$, (a, b)	closed, open intervals	123
	$S = T$	set equality	123
	$\emptyset$	empty (or null) set	124
	$S \subseteq T$	S is a subset of T	125
	$S \subset T$	S is a proper subset of T	126
	$\|S\|$	cardinality of S	127
	$\mathcal{P}(S)$	power set of S	128
	$(a_1, \ldots, a_n)$	n-tuple	128
	(a, b)	ordered pair	128
	$A \times B$	Cartesian product of A and B	129
	$A \cup B$	union of A and B	133
	$A \cap B$	intersection of A and B	134
	$A - B$	difference of A and B	135
	$\overline{A}$	complement of A	135
	$\bigcup_{i=1}^{n} A_i$	union of A_i, $i = 1, 2, \ldots, n$	140
	$\bigcap_{i=1}^{n} A_i$	intersection of A_i, $i = 1, 2, \ldots, n$	140
	$A \oplus B$	symmetric difference of A and B	145
	$\aleph_0$	cardinality of a countable set	180
	$\mathfrak{c}$	cardinality of $\mathbf{R}$	185



$A - B$ is shaded.

FIGURE 3 Venn diagram for the difference of A and B.



$\overline{A}$ is shaded.

FIGURE 4 Venn diagram for the complement of the set A.

2.2.2 Set Identities

Set identities and propositional equivalences are just special cases of identities for Boolean algebra.

Table 1 lists the most important identities of unions, intersections, and complements of sets. We will prove several of these identities here, using three different methods. These methods are presented to illustrate that there are often many different approaches to the solution of a problem. The proofs of the remaining identities will be left as exercises. The reader should note the similarity between these set identities and the logical equivalences discussed in Section 1.3. (Compare Table 6 of Section 1.6 and Table 1.) In fact, the set identities given can be proved directly from the corresponding logical equivalences. Furthermore, both are special cases of identities that hold for Boolean algebra (discussed in Chapter 12).

TABLE 1 Set Identities.

Identity	*Name*
$A \cap U = A$ $A \cup \emptyset = A$	Identity laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{(\overline{A})} = A$	Complementation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive laws
$\overline{A \cap B} = \overline{A} \cup \overline{B}$ $\overline{A \cup B} = \overline{A} \cap \overline{B}$	De Morgan's laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \emptyset$	Complement laws