

CSE390D—Introduction to Discrete Math

Sample Final

1. (14 points) The table below contains English descriptions in terms of x and y of relations on the set \mathbb{Z}^+ (x being related to y iff the described condition holds). Fill in the table indicating whether each relation is reflexive, symmetric, antisymmetric or transitive.

Relation	reflexive?	symmetric?	antisymmetric?	transitive?
$x + y = 2$				
$(x - 5)^2 = (y - 5)^2$				
$xy = 9$				
$2x = 3y$				
$2 \mid xy$				
$2 \mid x(y + 1)$				
$x \bmod y = 0$				

2. (8 points)
- (4 points) A certain high school has 145 women and 138 men in its graduating class. The graduating students select one man and one woman from their class for each of the following categories: most likely to succeed, most likely to be remembered, most scholarly, and most friendly. If nobody is allowed to win more than one category, how many different outcomes are possible?
 - (4 points) You are helping to organize a banquet for 38 people. Each person has chosen one of four entrées (chicken, beef, vegetarian, and fish). Your job is to tell the catering service how many of each entrée to bring. How many possible orders might you place with the catering company?
3. (8 points) Prove using mathematical induction that:
- $$3 + 7 + 11 + 15 + 19 + \cdots + (4n - 1) = 2n^2 + n$$
- Provide a formal definition for the overall proposition being proved ($P(n)$) using summation notation and indicate the domain for n . Then provide a proof by induction, clearly indicating where you are applying the inductive hypothesis.
4. (8 points) Genetic sequences are composed of the letters “A”, “G”, “T”, and “C”.
- (2 points) How many genetic sequences of length 10 have exactly two G’s and exactly two C’s?
 - (4 points) How many genetic sequences of length 10 have exactly two G’s or exactly two C’s (or both)?

- c. (2 points) If we count how many of each letter occur in a genetic sequence of length 10, how many different answers might we get (where an answer involves just the counts for each letter)?
5. (8 points) Prove that there are no three consecutive positive integers such that the cube of the third is the sum of the cubes of the first two.
6. (8 points) The Department of Homeland Security has commissioned several studies in an effort to develop their own version of gaydar. So far they have determined that 4% of the men in San Francisco are gay, 90% of the gay men in San Francisco say “That’s fierce” regularly, and 98% of the nongay men in San Francisco don’t say “That’s fierce” regularly.
 - a. (4 points) What is the probability that a man from San Francisco is gay given that he says “That’s fierce” regularly?
 - b. (4 points) What is the probability that a man from San Francisco is not gay given that he doesn’t say “That’s fierce” regularly?
7. (8 points)
 - a. (4 points) Suppose that a fair coin is flipped 20 times and we count how many times we get heads versus tails. What is the probability that the number of heads is not equal to the number of tails?
 - b. (4 points) Players are allowed to bet on the outcome of the flipping of a fair coin. The coin is flipped four different times. If two of the four flips come up heads, each player wins \$1. If three of the four flips come up heads, each player wins \$2. If all four flips come up heads, each player wins \$4. In all other cases the players win nothing. What is the expected amount won, on average, each time this game is played? Notice that you aren’t asked to take into account how much it costs to play this game, just how much a player can expect to win on average.
8. (8 points) Let R be the relation $\{(1, 2), (2, 3), (3, 1), (4, 2), (4, 5)\}$ defined on the set $\{1, 2, 3, 4, 5\}$.
 - a. (2 points) Draw the graph of R .
 - b. (2 points) Draw the graph of the symmetric closure of R .
 - c. (4 points) Draw the graph of the transitive closure of R .
9. (10 points) Prove or disprove that a number is irrational if its square is irrational.
10. (12 points) Assuming that the sum of two real-valued variables x and y is a constant c :

$$x + y = c$$

Prove that the product xy is maximal when $x = y$. You are not allowed to use calculus.

11. (8 points) A university needs to assign 180 students to 3 different dorms called LittleDorm, MediumDorm and BigDorm. LittleDorm houses 40 people, MediumDorm houses 60 people and BigDorm houses 80 people. Of the 180 students to be assigned to dorms, 65 are women and 115 are men.
- a. (2 points) How many ways are there to assign the 180 students to the 3 dorms?
 - b. (2 points) How many ways are there to assign the 180 students to the 3 dorms if LittleDorm is to be all men?
 - c. (2 points) How many ways are there to assign the 180 students to the 3 dorms if LittleDorm is to be all women and BigDorm is to be all men?
 - d. (2 points) Suppose that the university has given priority to some students to pick which dorm they want to be in. If 10 students have been allowed to pick LittleDorm, 20 students have been allowed to pick MediumDorm and 25 students have been allowed to pick BigDorm, how many ways are there to assign the remaining students to the 3 dorms?