

CSE390D—Introduction to Discrete Math
Homework #8 (probability, relations)
due: in class, Friday, 12/6/24

You are to complete the following problems.

1. What is the conditional probability that exactly four heads appear when a fair coin is flipped five times, given that the first flip came up tails?
2. Let E be the event that a randomly generated bit string of length three contains an odd number of 1s, and let F be the event that the string starts with 1. Are E and F independent (show your work).
3. Assume that the probability that a child is a boy is 0.51 and that the sexes born into a family are independent. What is the probability that a family of five children has:
 - a. Exactly three boys?
 - b. At least one boy?
 - c. At least one girl?
 - d. All children of the same sex?
4. When a test for steroids is given to soccer players, 98% of the players taking steroids test positive and 12% of the players not taking steroids test positive. Suppose that 5% of soccer players take steroids. What is the probability that a soccer player who tests positive takes steroids (using Bayes' Theorem)?

5. Determine whether the relation R on the set of all people is reflexive, symmetric, antisymmetric, and/or transitive where (a, b) is in R if and only if
- a. a is taller than b .
 - b. a and b were born on the same day.
 - c. a has the same first name as b .
 - d. a and b have a common grandparent.
6. Which of these relations on the set of all people are equivalence relations? Determine the properties of an equivalence relationship that the others lack.
- a. $\{(a, b) \mid a \text{ and } b \text{ are the same age}\}$
 - b. $\{(a, b) \mid a \text{ and } b \text{ have the same parents}\}$
 - c. $\{(a, b) \mid a \text{ and } b \text{ share a common parent}\}$
 - d. $\{(a, b) \mid a \text{ and } b \text{ have met}\}$
 - e. $\{(a, b) \mid a \text{ and } b \text{ speak a common language}\}$

Questions 1-5 below refer to the following information. Let S be the set of integers 1 through 8 inclusive. A pair of values (a, b) from S have the relation R if and only if $(a - b)$ is a prime number (recall that 0 and 1 are not prime).

1. Draw a directed graph representing R

2. Provide a zero-one matrix representing R

3. List the ordered pairs that would have to be added to form the reflexive closure of R

4. List the pairs that would have to be added to form the symmetric closure of R

5. List the pairs that would have to be added to form the transitive closure of R