CSE390D—Introduction to Discrete Math
Homework #7 (counting, probability)
due: in class, Friday, 11/22/24

You are to complete the following problems.

1. A croissant shop has plain croissants, cherry croissants, chocolate croissants, almond croissants, apple croissants, and broccoli croissants. How many ways are there to choose:

    a. A dozen croissants?

    b. Three dozen croissants?

    c. Two dozen croissants with at least two of each kind?

    d. Two dozen croissants with no more than two broccoli croissants?

    e. Two dozen croissants with at least five chocolate croissants and and at least three almond croissants?

    f. Two dozen croissants with at least one plain croissant, at least two cherry croissants, at least three chocolate croissants, at least one almond croissant, at least two apple croissants, and no more than three broccoli croissants?

2. How many different combinations of pennies, nickels, dimes, quarters, and half dollars can a piggy bank contain if it has 20 coins in it?

3. How many solutions are there to the equation
    x1 + x2 + x3 + x4 = 17,
    where x1, x2, x3, and x4 are nonnegative integers?

4. How many strings of 20-decimal digits are there that contain two 0s, four 1s,, three 2s, one 3, two 4s, three 5s, two 7s, and three 9s?

5. How many different strings can be made from the letters in AARDVARK, using all the letters, if all three As must be consecutive?

6. How many different bit strings can be formed using six 1s and eight 0s?

7. A professor packs her collection of 40 issues of a mathematics journal in four boxes with 10 issues per box. How many different ways can she distribute the journals if
    a. Each box is numbered, so that they are distinguishable?

    b. The boxes are identical, so that they cannot be distinguished?

8. What is the probability that a card selected from a deck is an ace or a heart?

9. What is the probability that a 5-card poker hand contains the ace of hearts?

10. What is the probability that a 5-card poker hand contains exactly one ace?

11. What is the probability that a positive integer not exceeding 100 selected at random is divisible by 3?

12. What is the probability that a player wins the prize offered for correctly choosing 5 (but not six) numbers out of six integers chosen at random from the integers between 1 and 40 inclusive?

All of the following questions involve a particular counting task. The board game Risk comes with a deck of 42 cards, one per country. Each card has one of three symbols printed on it: a foot-soldier, a canon or a soldier on horseback. The rules of Risk say that you can "turn in" 3 cards if you have 3 of any one kind or 1 of each kind. It turns out that once you have 5 cards, you're guaranteed to have one or the other way of turning in. All of the problems below involve 5-card hands. The cards within a hand are not ordered. All that matters is which cards are in the hand, not the order in which they appear.

The Risk deck of cards is the example used here, but we'll simplify it to make it easier to describe. Imagine we have a deck of 42 cards that are numbered from 1 to 42 (which means that no two cards are the same). Assume that 14 of the 42 cards have been marked with an "A", 14 with a "B" and 14 with a "C". Any one card has exactly one of these letters on it (no cards without letters, no cards with 2 or more letters). As in Poker, the order of the cards in a hand does not matter.

For the problems below, express your answer using combinations and permutations. Briefly describe each answer.

1. How many 5-card hands are possible?

2. How many 5-card hands have 5 of one letter?

3. How many 5-card hands have 4 of one letter and 1 of another?

4. How many 5-card hands have 3 of one letter and 2 of another?

5. How many 5-card hands have 3 of one letter, 1 of another letter and 1 of the third letter?

6. How many 5-card hands have 2 of one letter, 2 of another letter and 1 of the third letter?

note: The sum of your answers for questions 2 through 6 should equal your answer to question 1.

7. How many 5-card hands have exactly 2 A's?

8. How many 5-card hands have exactly 3 C's?

9. How many 5-card hands have exactly 5 A's or exactly 5 B's?

10. How many 5-card hands have exactly 1 A and exactly 1 B?

11. How many 5-card hands have at least 2 A's?

12. In how many 5-card hands is the number of A's plus the number of B's equal to 3?

13. How many 5-card hands have exactly 2 A's or exactly 2 B's?

14. How many 5-card hands have at least 2 A's and at least 2 B's?

15. How many 5-card hands have at least 1 B and at least 1 C?