CSE390D—Introduction to Discrete Math
Homework #6 (counting)
due: in class, Friday, 11/15/24

You are to complete the following problems.

1. How many positive integers less than 1000
    a. Are divisible by 7?
    b. Are divisible by 7 but not by 11?
    c. Are divisible by both 7 and 11?
    d. Are divisible by either 7 or 11?
    e. Are divisible by exactly one of 7 and 11?
    f. Are divisible by neither 7 nor 11?
    g. Have distinct digits?
    h. Have distinct digits and are even?

2. A bowl contains 10 red balls and 10 blue balls. A woman selects balls at random without looking at them.
    a. How many balls must she select to be sure of having at least three balls of the same color?
    b. How many balls must she select to be sure of having at least three blue balls?

3. How many ordered pairs of integers (a, b) are needed to guarantee that there are two ordered pairs (a_1, b_1) and (a_2, b_2) such that $a_1 \bmod 5 \equiv a_2 \bmod 5$ and $b_1 \bmod 5 \equiv b_2 \bmod 5$?

4. Show that if seven integers are selected from the first 10 positive integers, there must be at least two pairs of these integers with the sum 11. Is the conclusion true if six integers are selected rather than seven?

5. Find an increasing subsequence of maximal length and a decreasing subsequence of maximal length in the sequence 22, 5, 7, 2, 23, 10, 15, 21, 3, 17.

6. A coin is flipped eight times where each flip comes up either heads or tails. How many possible outcomes:
    a. Are there in total?
    b. Contain exactly two heads?
    c. Contain at most three tails?
    d. Contain the same number of heads and tails?
7. Seven women and nine men are on the faculty in the mathematics department at a school.
    a. How many ways are there to select a committee of five members of the department if at least one woman must be on the committee?
    b. How many ways are there to select a committee of five members of the department if at least one woman and at least one man must be on the committee?

8. How many strings of six lowercase letters from the English alphabet contain:
    a. The letter a?
    b. The letters a and b?
    c. The letters a and b in consecutive positions with a preceding b, with all the letters distinct?
    d. The letters a and b, where a is somewhere to the left of b in the string, with all the letters distinct?