CSE390D—Introduction to Discrete Math
Homework #4 (primes, sets)
due: in class, Friday, 10/25/24

You are to complete the following problems.

1. Find an inverse of 3 modulo 17 (a value that when multiplied by 3 is 1).

2. Solve the congruence $2x \equiv 7 \pmod{17}$.

3. Show that the positive integers less than 11, except 1 and 10, can be split into pairs of integers such that each pair consists of integers that are inverses of each other modulo 11.

4. Use the answer to problem 3 to show that $10! \equiv -1 \pmod{11}$.

5. Use Fermat's Little Theorem to compute 3^{302} mod 5, 3^{302} mod 7, and 3^{302} mod 11. Show your work.

6. Suppose that A = {2, 4, 6}, B = {2, 6}, C = {4, 6} and D = {4, 6, 8}. Determine which of these sets is are subsets of which other of these sets.

7. Let A = {a, b, c}, B = {x, y}, and C = {0, 1}. List all set members of:
   a. $A \times B$ x C
   b. C x B x A
   c. C x A x B
   d. B x B x B

8. Let A = {a, b, c, d, e}, B = {a, b, c, d, e, f, g, h}. List all set members of:
   a. $A \cup B$
   b. $A \cap B$
   c. A – B
   d. B – A

9. Find the sets A and B A and B if A – B = {1, 5, 7, 8}, B – A = {2, 10} and $A \cap B$ is {3, 6, 9}/

10. Determine whether each of these functions from **Z** to **Z** is one-to-one.
    a. f(n) = n – 1
    b. $f(n) = n^2 + 1$
    c. $f(n) = n^3$
    d. f(n) = ceiling(n/2) (ceiling is smallest integer less than or equal to a value)

11. Determine whether f: **Z** x **Z** → **Z** is onto if
    a. f(m, n) = 2m – n.
    b. $f(m, n) = m^2 - n^2$
    c. f(m, n) = m + n + 1
    d. f(m, n) = |m| - |n|
    e. $f(m, n) = m^2 - 4$

12. Determine whether each of these functions is a bijection from **R** to **R**.

   a. $f(x) = -3x + 4$
   b. $f(x) = -3x^2 + 7$
   c. $f(x) = x^5 + 1$

13. Prove or disprove that $n^2 + 3n + 1$ is always prime for integers $n > 0$.

14. Prove or disprove that for any prime $p > 3$, $p \equiv 1 \pmod{6}$ or $p \equiv 5 \pmod{6}$.