CSE390D—Introduction to Discrete Math
Homework #3 (mathematical reasoning, number theory)
due: in class, Friday, 10/18/24

You are to complete the following problems. In addition, you are to complete the 10 problems that appear after the numbered problems.

1. Show that if a, b, c, and d are integers such that a | c and b | d, then ab | cd.

2. Show that if $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, where a, b, c, d, and m are integers with $m \geq 2$, then $a - c \equiv b - d \pmod{m}$.

3. Prove that if n is an odd positive integer, then $n^2 \equiv 1 \pmod{8}$.

4. Show that if a, b, c, and m are integers such that $m \geq 2$, $c > 0$, and $a \equiv b \pmod{m}$, then $ac \equiv bc \pmod{mc}$.

5. Determine whether the integers in each of these sets are pairwise relatively prime.
   a. 21, 34, 55
   b. 14, 17, 85
   c. 25, 41, 49, 64
   d. 17, 18, 19, 23

6. What are the greatest common divisors of these pairs of integers?
   a. $2^2 \cdot 3^3 \cdot 5^5$, $2^5 \cdot 3^3 \cdot 5^2$
   b. $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13$, $2^{11} \cdot 3^9 \cdot 11 \cdot 17^{14}$
   c. 17, 17^{17}
   d. $2^2 \cdot 7$, $5^3 \cdot 13$
   e. 0, 5
   f. $2 \cdot 3 \cdot 5 \cdot 7$, $2 \cdot 3 \cdot 5 \cdot 7$

7. If the product of two integers is $2^7 3^8 5^2 7^{11}$ and their greatest common divisor is $2^3 3^4 5$, what is their least common multiple?

8. Show that if a, b, and m are integers such that $m \geq 2$ and $a \equiv b \pmod{m}$, then gcd(a, m) = gcd(b, m).

9. Use the Euclidean algorithm to find
   a. gcd(1, 5).
   b. gcd(100, 101).
   c. gcd(123, 277).
   d. gcd(1529, 14039)
   e. gcd(1529, 14038)
   f. gcd(11111, 111111)

Assume that the following statements are true:

1. Sam has a high IQ.
2. John likes all people with high IQs.
3. Fred is a genius.
4. You can join Mensa only if you're a genius.
5. Sarah likes Kip.
6. Sarah does not like anyone whom John likes.
7. Only people with high IQs are geniuses.
8. John doesn't like Phil.
9. Sarah does not like Mary.
10. Mike does not have a high IQ.

For each statement below, indicate "yes" or "no" whether the statement can be concluded from the facts above . If it can be concluded, list all of the statements above that are needed to draw the conclusion (use the statement numbers to refer to them).

1. Sarah likes Phil.

2. John likes Fred.

3. Sam is a genius.

4. John does not like Mike.

5. Mike is not a genius.

6. Sarah does not like Fred.

7. Kip does not have a high IQ.

8. Mary has a high IQ.

9. John likes Mary

10. Phil cannot join Mensa.