CSE390D—Introduction to Discrete Math
Homework #2 (nested quantifiers, inference, and proofs)
due: in class, Friday, 10/11/24

You are to complete the following problems. In doing so, you should make use of the following definition.

> The integer n is even if there exists an integer k such that n = 2k, and n is odd if there exists an integer k such that n = 2k + 1. (Note that an integer is either even or odd, and no integer is both even and odd).

1. Let F(x, y) be the statement "x can fool y," where the domain consists of all people in the world. Use quantifiers to express each of these statements.
   a. Everybody can fool Fred.
   b. Evelyn can fool everybody.
   c. Everybody can fool somebody.
   d. There is no one who can fool everybody.
   e. Everyone can be fooled by somebody.
   f. No one can fool both Fred and Jerry.
   g. Nancy can fool exactly two people.
   h. There is exactly one person whom everybody can fool.
   i. No one can fool himself or herself.
   j. There is someone who can fool exactly one person besides himself or herself.

2. Let I(x) be the statement "x has an Internet connection" and C(x, y) be the statement "x and y have chatted over the Internet," where the domain for the variables x and y consists of all students in your class. Use quantifiers to express each of these statements.
   a. Jerry does not have an Internet connection.
   b. Rachel has not chatted over the Internet with Chelsea.
   c. No one in the class has chatted with Bob.
   d. Sanjay has chatted with everyone except Joseph.
   e. Someone in your class does not have an Internet connection.
   f. Exactly one student in your class has an Internet connection.
   g. Everyone except one student in your class has an Internet connection.
   h. Someone in your class has an Internet connection but has not chatted with anyone else in your class.
   i. There are two students in your class who have not chatted with each other over the Internet.
   j. There is a student in your class who has chatted with everyone in your class over the Internet.
   k. There are two students in the class who between them have chatted with everyone else in the class.

3. Translate each of these nested quantifications into an English statement that expresses a mathematical fact. The domain in each case consists of all real numbers.
   a. $\exists x \forall y(x + y = y)$
   b. $\forall x \forall y(((x \geq 0) \wedge (y < 0)) \rightarrow (x - y > 0))$
   c. $\forall x \forall y((x \neq 0) \wedge (y \neq 0) \leftrightarrow (xy \neq 0))$

4. What rule of inference is used in each of these arguments?
   a. Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.
   b. It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous.
   c. Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard.
   d. Steve will work at a computer company this summer. Therefore, this summer Steve will work at a computer company or he will be a beach bum.
   e. If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material.

5. For each of these arguments determine whether the argument is correct or incorrect and explain why.
   a. Everyone enrolled in the university has lived in a dormitory. Mia has never lived in a dormitory. Therefore, Mia is not enrolled in the university.
   b. A convertible car is fun to drive. Isaac's car is not a convertible. Therefore, Isaac's car is not fun to drive.
   c. Quincy likes all action movies. Quincy likes the movie *Eight Men Out*. Therefore, *Eight Men Out* is an action movie.
   d. All lobstermen set at least a dozen traps. Hamilton is a lobsterman. Therefore, Hamilton sets at least a dozen traps.

6. Use a direct proof to show that the product of two odd numbers is odd.

7. Prove that if m and n are integers and mn is even, then m is even or n is even (you may apply your result from problem #6).

8. Prove that if n is an integer and 3n + 2 is even, then n is even using
   a. a proof by contraposition.
   b. a proof by contradiction.

9. Prove that if n is a positive integer, then n is even if and only if 7n + 4 is even.