CSE390D—Introduction to Discrete Math Homework #1 (propositional and predicate logic) due: in class, Friday, 10/4/24

You are to complete the following problems.

1. Let p and q be the propositions

p: I bought a lottery ticket this week.

q: I won the million dollar jackpot.

Express each of these propositions as an English sentence.

b. $p \lor q$

- c. $p \rightarrow q$
- d. $p \wedge q$
- e. $p \leftrightarrow q$

f.
$$\neg p \rightarrow \neg q$$

g.
$$\neg p \land \neg q$$

- h. $\neg p \lor (p \land q)$
- 2. Let p, q, and r be the propositions
 - p: You have the flu.
 - q: You miss the final examination.
 - r: You pass the course.

Express each of these propositions as an English sentence.

- a. $p \rightarrow q$
- b. $\neg q \leftrightarrow r$
- c. $q \rightarrow \neg r$
- d. $p \lor q \lor r$

e.
$$(p \rightarrow \neg r) \lor (q \rightarrow \neg r)$$

- f. $(p \land q) \lor (\neg q \land r)$
- 3. Let p, q, and r be the propositions
 - p: You get an A on the final exam.

q: You do every exercise in this book.

r: You get an A in this class.

Write these propositions using p, q, and r, and logical connectives (including negations)

- a. You get an A in this class, but you do not do every exercise in this book.
- b. You get an A on the final, you do every exercise in this book, and you get an A in this class.
- c. To get an A in this class, it is necessary for you to get an A on the final.
- d. You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.
- e. Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.
- f. You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

- 4. Let C(x) be the statement "x has a cat," let D(x) be the statement "x has a dog," and let F(x) be the statement "x has a ferret." Express each of these statements in terms of C(x), D(X), F(x), quantifiers, and logical connectives. Let the domain consist of all students in your class.
 - a. A student in your class has a cat, a dog, and a ferret.
 - b. All students in your class have a cat, a dog, or a ferret.
 - c. Some student in your class has a cat and a ferret, but not a dog.
 - d. No student in your class has a cat, and dog, and a ferret.
 - e. For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet.
- 5. Let Q(x) be the statement "x + 1 > 2x." If the domain consists of all integers, what are these truth values (answers are true or false)?
 - a. Q(0)
 - b. Q(-1)
 - c. Q(1)
 - d. $\exists x Q(x)$
 - e. $\forall xQ(x)$
 - f. $\exists x \neg Q(x)$
 - g. $\forall x \neg Q(x)$
- 6. Determine the truth value of each of these statements if the domain consists of all real numbers. Provide an example when appropriate.
 - a. $\exists x(x^3 = -1)$
 - b. $\exists x(x^4 < x^2)$
 - c. $\forall x((-x)^2 = x^2)$
 - d. $\forall x(2x > x)$
- 7. Let P(x), Q(x), R(x), and S(x) be the statements "x is a duck," "x is one of my poultry," "x is an officer," and "x is willing to waltz," respectively. Express each of these statements using quantifiers, logical connectives, and P(x), Q(x), R(x) and S(x).
 - a. No ducks are willing to waltz.
 - b. No officers ever decline to waltz.
 - c. All my poultry are ducks.
 - d. My poultry are not officers.
 - e. Does (d) follow from (a), (b), and (c)? If not, is there a correct conclusion?