CSE 390B: Building Academic Success Through Bottom-Up Computing Spring 2023

Midterm Exam Solutions May 4th, 2023, at 2:30pm

Name: _____

UW NetID: _____

**Instructions:**

- Make sure you have included your name (first & last) and your UW NetID on this page.
- When you finish the exam, turn in your exam to the course staff.
- You will have 60 minutes to complete the exam.
- Questions are not necessarily in order of difficulty.
- This exam is closed-note, closed-book (except for the reference sheet).
- This exam contains 100 points distributed unevenly among five questions (some with multiple parts).

**Advice:**

- Read each question carefully. Understand a question before you start writing.
- When applicable, elaborate on your answer, explain your thought process, and write down the intermediate steps for possible partial credit. However, clearly indicate what your final answer is.
- The questions are not necessarily in order of difficulty. Feel free to skip around. Do your best to get to all the questions.
- If you have a question, please raise your hand, and the course staff will get to you shortly.
- Take deep breaths and relax. Remember that you are here to learn.

**Technical Details:**

- You may specify Boolean operations using symbols or words (e.g., for the And gate, you may use the symbol & or “And”).
- When using a Mux or DMux gate, explicitly show or describe the select bits that the inputs are connected to (e.g., the a input of the Mux is connected to the select bit of 0).

| Question | 1 | 2 | 3 | 4 | 5 | Total |
|---|---|---|---|---|---|---|
| Possible Points | 25 | 10 | 20 | 20 | 25 | 100 |

1. (25 points) In this problem, you will build Boolean circuits with three inputs and one output. You may only use two-input And and Or gates and single-input Not gates.

   A palindrome is a sequence of characters that is the same backward as is it is forward. For example, the words “kayak” and “Hannah” are palindromes because when you reverse the characters in that word, the word ends up being the same as the original word. Similarly, a sequence of binary bits is a palindrome if the bits are in the same order in reverse as its original order. For example, “10101” is a palindrome while “10111” is not a palindrome. For the problem below, determine if the sequence of bits in the order {a, b, c} represents a palindrome.

   a. Given the specification of IsPalindrome described in the paragraph above, fill in the output of the truth table for the circuit with three inputs and one output.

| a | b | c | isPalindrome |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

   b. Based on the truth table you filled out, write a Boolean expression for isPalindrome.

      isPalindrome = (~a & ~b & ~c) | (~a & b & ~c) | (a & ~b & c) | (a & b & c)

c. Implement the Boolean expression you came up with from part b. for the out output by completing the following HDL template or drawing a circuit diagram. You do not need to do both.

```
CHIP IsPalindrome {
      // a, b, and c represent the bits in binary
      IN a, b, c;

      // isPalindrome is 1 if the result of the binary bits
      // in the order {a, b, c} is a palindrome, 0 otherwise
      OUT isPalindrome;

      PARTS:
      // Your code or circuit diagram here:
      // Setting up nots of the input bits
      Not (in=a, out=nota);
      Not (in=b, out=notb);
      Not (in=c, out=notc);

      // Row 1 Boolean expression
      And (a=nota, b=notb, out=nota-notb);
      And (a=nota-notb, b=notc, out=out1);

      // Row 3 Boolean expression
      And (a=nota, b=b, out=nota-b);
      And (a=nota-b, b=not-c, out=out2);

      // Row 6 Boolean expression
      And (a=a, b=notb, out=a-notb);
      And (a=a-notb, b=c, out=out3);

      // Row 8 Boolean expression
      And (a=a, b=b, out=ab);
      And (a=ab, b=c, out=out4);

      // Combining all the Boolean expressions using Or
      Or (a=out1, b=out2, out=out1-or-out2);
      Or (a=out3, b=out5, out=out3-or-out4);
      Or (a=out1-or-out2, b=out3-or-out4, out=out);




}
```

2. (10 points) Free response questions. Describe the answers to the following questions in a paragraph.

    a. List the two ways to update the A register in the Hack computer and give an example of each. Why are there two separate ways to update the A register as opposed to just one option?

    The two ways to update the A register in the Hack computer is using the A-instruction and the C-instruction.

    Example using the A-instruction: @5

    Example using the C-instruction: A = D

    There are many reasons there are two separate ways to update the A register. First, without the A-instruction, it would be inconvenient to perform an operation with a numerical constant (e.g., D = D + 100). With the A-instruction, you can set the A register to 100 and then perform D = D + A as opposed to D = D + 1 a hundred times. Second, without the A-instruction, it wouldn't be possible to make jumps to certain locations of Hack Assembly code, since using labels and symbols depends on the A-instruction. Lastly, without the C-instruction method of updating the A register, it wouldn't be possible to use a value in memory to address a location on memory.

    b. Describe in your own words how the Hack computer determines whether a **conditional jump** should be performed given the output control bits of the ALU (ng and zr) and the jump bits of the C-instruction (j1, j2, and j3). Give an example of the values of ng, zr, j1, j2, and j3 that would cause a jump to occur and explain why in 2-3 sentences.

    The ALU output control bits of ng and zr tell us whether the number computed from the ALU is negative, zero, or positive. The j1, j2, and j3 bits tell us the inequality or equality check to be performed against the ALU's computational output that determines whether or not we should perform a jump. For example, if ng is 1 and zr is 0, indicating that the ALU's output is negative, then we would jump is j1 is 1, since j1 specifies that we should jump if the ALU's output is less than zero.

3. (20 points) In this problem, you may only use two-input Mux gates, DFFs, and combinational logic gates.

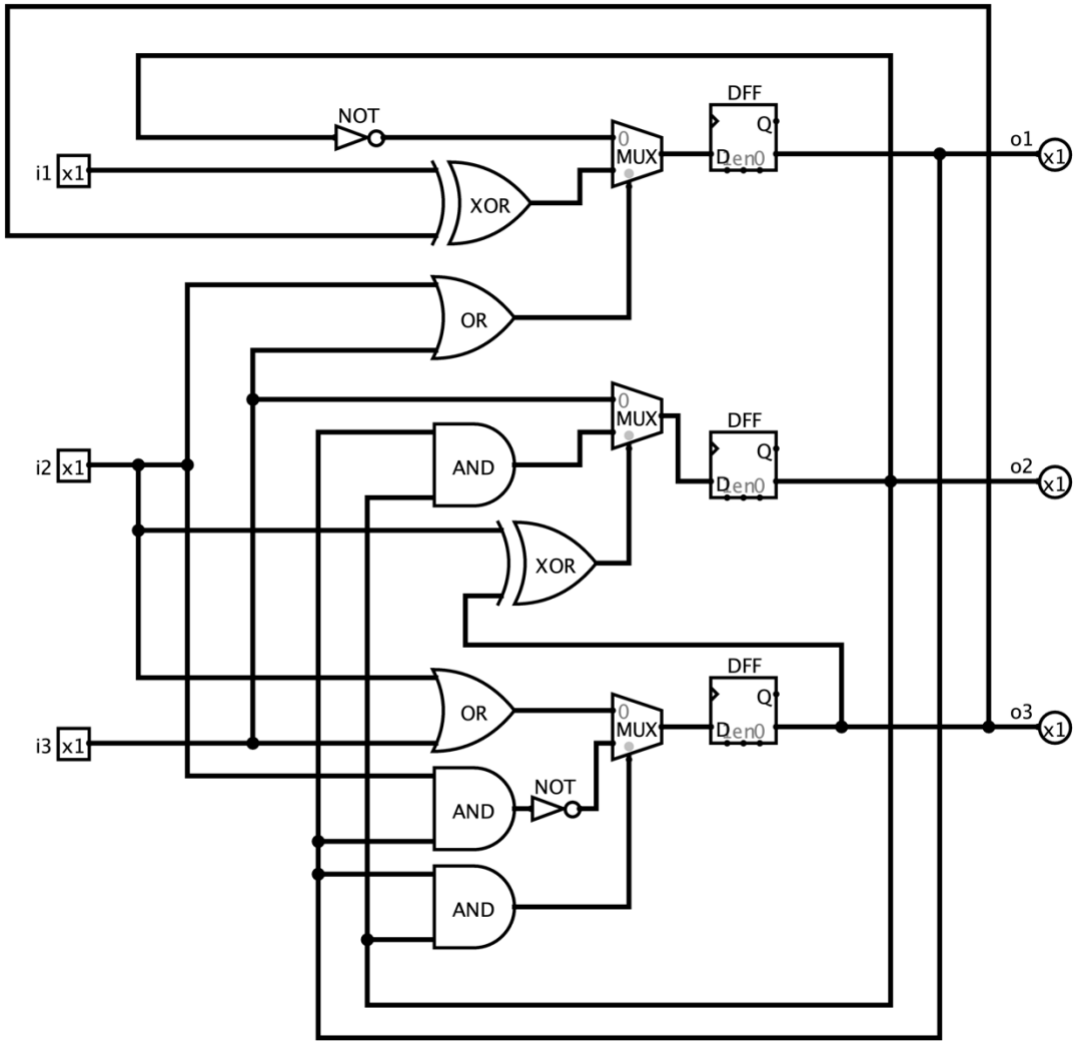
   Draw the following circuit specification using conventional notation, omitting implicit clock signals.

   - The circuit takes three data inputs (**`i1`**, **`i2`**, and **`i3`**)
   - The circuit has three outputs (**`o1`**, **`o2`**, and **`o3`**)
   - Each output at time **`t + 1`** is defined as follows:

     ```
     ○ if (i2 | i3):          o1(t + 1) = ~o2
       else:                  o1(t + 1) = i1 ^ o3

     ○ if (i1 ^ o3):          o2(t + 1) = o1 & o2
       else:                  o2(t + 1) = i3

     ○ if (o1 & o2):          o3(t + 1) = ~(o1 & i2)
       else:                  o3(t + 1) = i2 | i3
     ```



4. (20 points) Below is a sample program written in high-level pseudocode. Write an equivalent Hack Assembly program using the virtual registers R0, R1, R2, and R3, each of which corresponds to the values in memory at addresses 0, 1, 2, and 3, respectively.

```
if (-R0 – R2 != 13) {
      R0 = -3
} else if (R2 | R3 <= 20) {
      R0 = -R1
} else {
      R0 = R2 + R3
}
```

```
@R0
D = -M
@R2
D = D - M
@13
D = D - A
@BRANCH1
D; JNE
@R2
D = M
@R3
D = D | M
@20
D = D – A
@BRANCH2
D; JLE
@R2
D = M
@R3
D = D + M
@R0
M = D
@END
0; JMP
(BRANCH1)
   @R0
   M = -1
   M = M - 1
   M = M - 1
   @END
   0; JMP
(BRANCH2)
   @R1
   D = -M
   @R0
   M = D
(END)
   @END
   0; JMP
```

5. (25 points) Below is a Hack Assembly program with a bug.

```
01.     (START)
02.         @R0
03.         M = 0
04.         @2
05.         D = A
06.         @R1
07.         M = D
08.     (LOOP)
09.         @R1
10.         D = M
11.         @8
12.         D = D - A
13.         @END
14.         D; JGE
15.     (CHECK_EQUAL_ONE)
16.         @R1
17.         A = M
18.         D = M + 1
19.         // PART I
20.         @UPDATE_INDEX
21.         D; JNE
22.         @R0
23.         M = M + 1
24.     (UPDATE_INDEX)
25.         @R1
26.         M = M + 1
27.         // PART II
28.         @LOOP
29.         0; JMP
30.     (END)
31.         @END
32.         0; JMP
```

Here is the memory state before the Hack Assembly code to the left runs (we will use this to answer later parts of this problem):

| **Address** | **Value** |
|---|---|
| 0 | 1 |
| 1 | 1 |
| 2 | 1 |
| 3 | 6 |
| 4 | 9 |
| 5 | -1 |
| 6 | 4 |
| 7 | 5 |

a. Trace through the code starting with the state of memory given in the table. Indicate the value of the registers D, A, and M at each of the following locations commented with “PART #” the first time you reach that location when executing the code.

   i. Values of D, A, and M when first reaching comment with “PART I”

      D = 2

      A = 2

      M = 1

   ii. Values of D, A, and M when first reaching comment with “PART II”

      D = 2

      A = 1

      M = 3

b. Starting with the state of memory given in the table, what are the values stored at address 0, address 1, and address 2 in memory after the Hack Assembly code runs to completion (i.e., enters the END infinite loop)?

      Value at address 0 = 1

      Value at address 1 = 8

      Value at address 2 = 1

c. The Hack Assembly code is supposed to count the number of elements stored in memory addresses R2 to R7 that equal 1 and store the result in R0. The Hack Assembly program above attempts to be equivalent to the following pseudocode:

```
1.      R0 = 0
2.      for (R1 = 2; R1 < 8; R1++) {
3.          if (RAM[R1] == 1) {
4.              R0 += 1
5.          }
6.      }
```

We can fix the bug in the Hack Assembly code by **modifying** a single line of Hack Assembly. Circle the section of code indicated by the symbols in the Hack Assembly program in which we should modify the line to fix the bug.

**START** **LOOP** **CHECK_EQUAL_ONE** **UPDATE_INDEX** **END**

d. What line number would you modify to fix the bug in the Hack Assembly program, and what should the line of code be instead?

i. Line number: 18

ii. Line of Hack Assembly to fix the bug: `D = M - 1`

Does the buggy Hack Assembly program return the correct output given the initial memory state shown in the table? If so, how could you change the initial memory state for the bug to appear in the output? If not, how could you change the initial memory state for the buggy assembly program to produce the correct output?

Yes, the buggy Hack Assembly program does return the correct output given the initial memory state show in the table. This is because there is a single 1 value in memory, but the program actually counts the single -1 value in memory. To cause the bug to appear, change the number of occurrences of -1 in memory from addresses 2-7 to any number besides one.