CSE 374: Lecture 24

Week 9: hexadecimal and number storage

Number systems and BASE

Generally use base 10

(10 fingers)

234

$2x100 + 3x10 + 4x1$

$2x10^2 + 3x10^1 + 4x10^0$

Digital systems - base 2

(binary)

234 = 0b11101010

$1x2^7 + 1x2^6 + 1x2^5 + 0x2^4 +$
$1x2^3 + 0x2^2 + 1x2^1 + 0x2^0$

Base 16 - very compact

(hexadecimal)

234 = 0xEA

$14x16^1 + 10x16^0$

Need 16 digits,
so we used [0-9A-F]

Notice: 234 takes 3 digits to express in base 10, 8 in base 2, and 2 in base 16.

Integer representations

Digital systems are ‘on’ or ‘off’, thus, Binary.

- The hardware (and C) supports two flavors of integers
  - unsigned – only the non-negatives
  - signed – both negatives and non-negatives
- There are only 2W distinct bit patterns of W bits, so...
  - Cannot represent all the integers
  - Unsigned values: $0 \ldots 2^{W}-1$ **<= $2^{4}-1$ -> 1111 -> $2^{3}+2^{2}+2^{1}+2^{0}$ -> 8+4+2+1 -> 15**
  - Signed values: $-2^{W}-1 \ldots 2^{W-1} -1$
- Reminder: terminology for binary representations

“Most-significant” / “high-order” bit(s) “Least-significant” / “low-order” bit(s)

0010010110101011

Signed Ints (obvious solution)

4 bit signed int

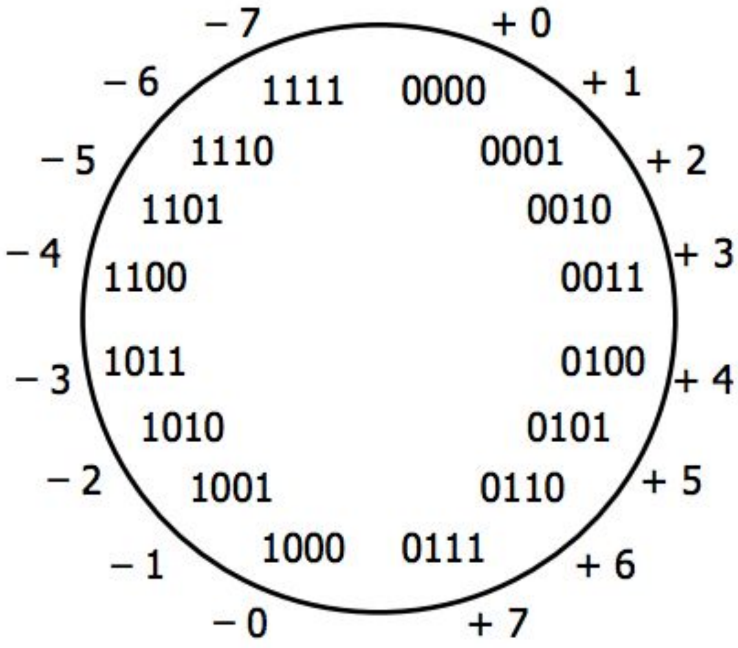
- Most significant bit is reserved for the sign
- Changes the range to $[-2^{w}-1, 2^{w-1} -1]$

Adding unsigned ints : (add and carry normally)

```
 0101
+0011
------
 1000
```

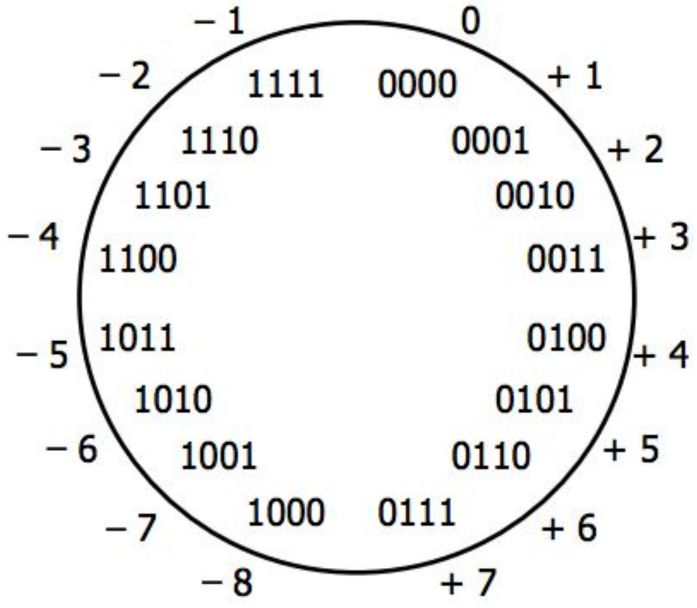
Adding signed ints : (gets tricky - notice `4-3 != 4+-3`)

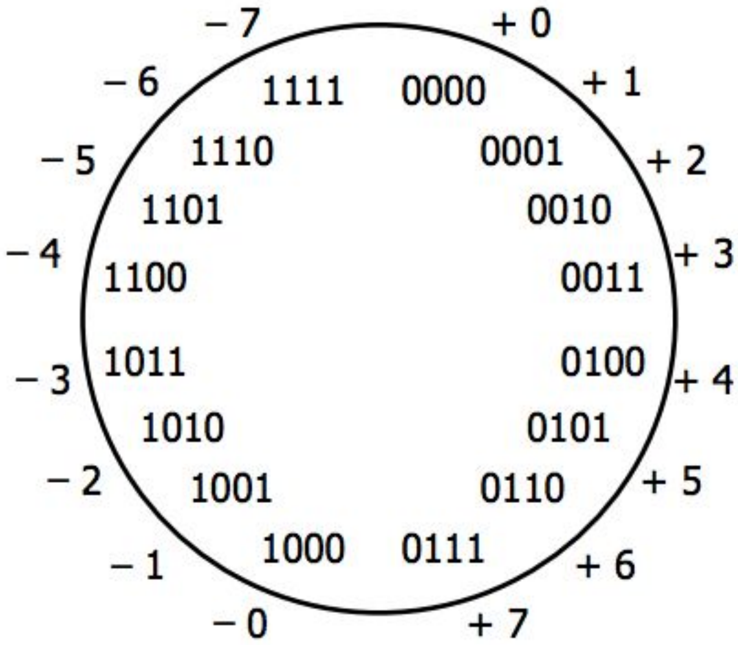
```
 0100
+1011
------
 1111    = 15
```



Twos-complement

Old version - notice the two different representations of ‘0’





Imagine the first bit is ‘subtract’ the value of the digit

Twos-complement: Benefits

Only 1 representation of 0

Most-significant bit is still the sign

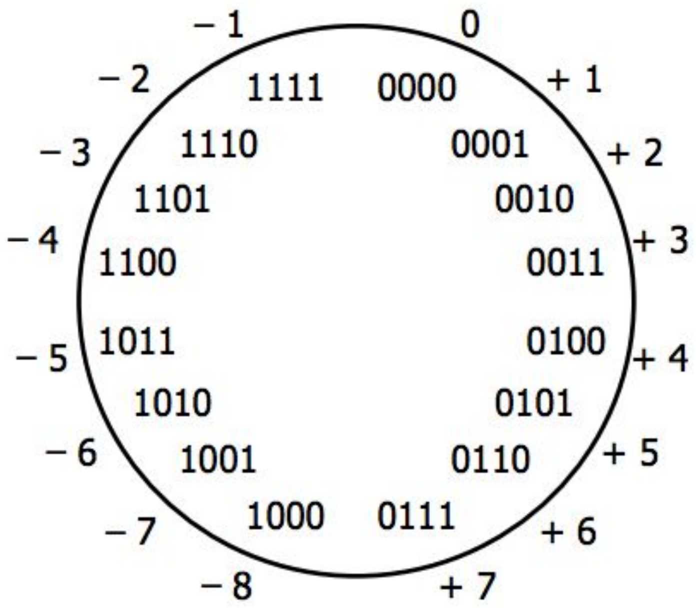
Negate a value

Bitwise complement + 1

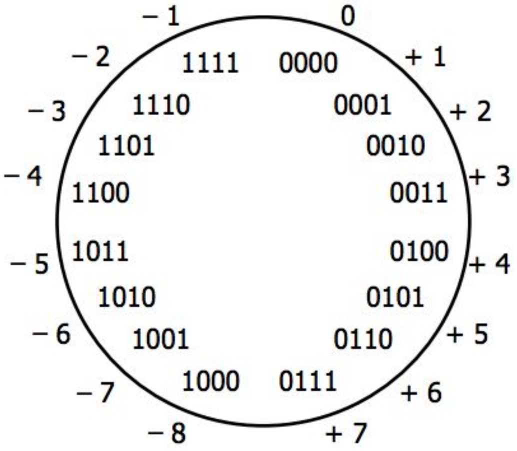
`0101  =  1010 + 1 = 1011`

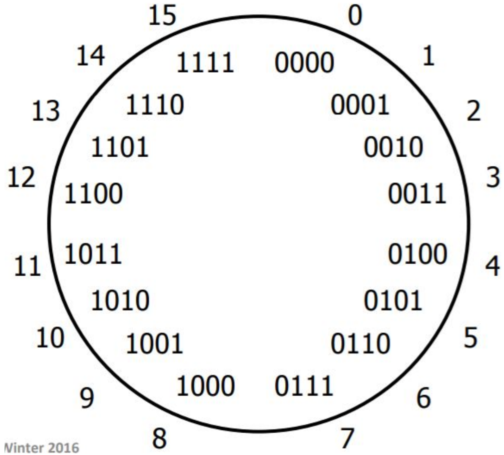
Adding becomes easy again:

`(4 - 3 = 4 + -3 = 1)`
`0100 + 1101 = 0001`



Twos-complement and unsigned ints





Get the two-complement number by subtracting 2^w from the unsigned number of the same representation:

`11(1011)-16 = -5(1011)`

Use the same algorithm for addition, so hardware implementation is simpler.

What happens if you ‘overflow’

Overflow: have numbers too big or small for your number of digits.

(Remember, using 4 bits, unsigned = [0,15] and signed [-8,7]

6+4 = ? (signed)

15+2 = ? (unsigned)

-6 - 6 = ? (signed)

2 - 6 = ? (unsigned)

Notes: You may get a warning for overflow with two-complement numbers, but probably not with unsigned numbers.

```
 0110
+0100
------
 1010 (-6!)
```

```
 1111
+0010
------
 0001 (1!)
```

```
 1010
-1010
------
 0100 (4!)
```

```
 0001
-0110
------
 1001 (9!)
```

C: ‘int’ and ‘unsigned’

```
int tx, ty;
unsigned ux, uy;
```

Explicit casting between signed & unsigned:

```
tx = (int) ux;
uy = (unsigned) ty;
```

Implicit casting also occurs via assignments and function calls:

```
tx = ux;
uy = ty;
```

The gcc flag *-Wsign-conversion* produces warnings for implicit casts, but *-Wall* does not!

Casting - doesn’t change underlying bits, they just get interpreted differently!

Note: in C, constants are assumed to be signed, unless the ‘U’ suffix is used: 15U -> 15 unsigned

Float Point Numbers

- Fractional binary numbers work in the same fashion as fractional decimal numbers
  - $1.25 = 1 \cdot 10^{0} + 2 \cdot 10^{-1} + 5 \cdot 10^{-2}$
  - $0b1.01 = 1 \cdot 2^{0} + 0 \cdot 2^{-1} + 1 \cdot 2^{-2} = 1 + 1/4 = 1.25$
- can have repeating just like decimal
  - 1/10 = 0b0.0001100110011[0011]…
- floating point values only represent numbers that can be written $x \cdot 2^{y}$
- like scientific notation
  - not 0b0.000101 but $1.01 \cdot 2^{4}$
- Floating point standard established
  - 1985, IEEE 754 - before that every system had a different approach

Floating Point Numbers

- Numerical form: V10 = **(–1)s * M * 2E**
  - Sign bit **s** determines whether number is negative or positive
  - Significand (mantissa) **M** normally a fractional value in range [1.0,2.0)
  - Exponent **E** weights value by a (possibly negative) power of two

`s`	`E: encodes exponent`	`M: encodes fraction`

Floating Point Numbers

- Numerical form: V10 = **(–1)s * M * 2E**

`s`	`E: encodes exponent`	`M: encodes fraction`

- For single precision (32 bits), we have s = 1 bit, E = 8 bits, M = 23 bits
- For double precision (64 bits), we have s = 1 bit, E = 11 bits, M = 52 bits
- Since we have a finite number of bits, some values will have to be approximated
- Special values
  - zero: s == 0, E == 0, M == 0
  - +∞, -∞: E == all ones, M == 0
  - NaN (not a number): E = all ones, M != 0
  - special values can pollute numerical computation

Floating Point Numbers

- As with integers, floats suffer from the fixed number of bits available to represent them
    - Can get overflow/underflow, just like ints
- Some “simple fractions” have no exact representation (e.g., 0.2)
- Can also lose precision, unlike ints “Every operation gets a slightly wrong result”
- Mathematically equivalent ways of writing an expression may compute different results
- Violates associativity/distributivity
- Never test floating point values for equality!
- Careful when converting between ints and floats!

Floating Points in C

- C offers two levels of precision
  - float single precision (32-bit)
  - double double precision (64-bit)
- `#include <math.h>` to get INFINITY and NAN constants
- Equality (==) comparisons between floating point numbers are tricky
  - often return unexpected results
  - Just avoid them!

Data type conversions

- Conversions between data types:
  - Casting between int, float, and double changes the bit representation.
- int → float
  - May be rounded: overflow not possible
- int → double or float → double
  - Exact conversion (32-bit ints; 52-bit frac + 1-bit sign)
- long int → double
  - Rounded or exact, depending on word size
- double or float → int
  - Truncates fractional part (rounded toward zero)
  - E.g. 1.999 -> 1, -1.99 -> -1
- “Not defined” when out of range or NaN: generally sets to Tmin

What about Hexadecimal?

Generally use base 10

(10 fingers)

234

2x100 + 3x10 + 4x1

$2x10^2 + 3x10^1 + 4x10^0$

Digital systems - base 2

(binary)

234 = 0b11101010

$1x2^7 + 1x2^6 + 1x2^5 + 0x2^4 +$
$1x2^3 +0x2^2+ 1x2^1 + 0x2^0$

Base 16 - very compact

(hexadecimal)

234 = 0xEA

$14x16^1 + 10x16^0$

Need 16 digits,
so we used [0-9A-F]

Computer represent things in binary. However, we can capitalize on different representations for compact storage, or for particular needs. One hexadecimal digit takes precisely 4 bits (one nibble) to store. Because 16 corresponds to 2 bytes conversion from binary to hexadecimal is convenient.

Hexadecimal in C

There is no unique type for hexadecimal in C. We use ‘unsigned int’ or ‘unsigned char’.

Remember, sizeof(int) = 2 or 4 [bytes]

and sizeof(char) = 1 [byte]

An unsigned char can hold values up to 255 or 0xFF (maximum two digit hex value)

```
unsigned char ahexvalue = 0xFE;
uintptr_t mymem = (uintptr_t) malloc (16);
for (int i = 0; i < 16; i++) {
    *((unsigned char*)(mymem+i)) = 0xFE;
}
```