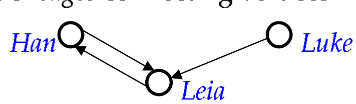
CSE 373: Graphs

Chapter 9

Graphs

Graphs:

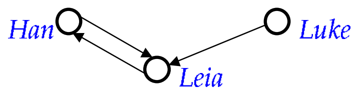
- A data structure useful for representing relationships between things
- A graph G is represented as $G = (V,E)$
  - V = a set of *vertices* (*nodes*)
  - E = a set of *edges* connecting vertices from V



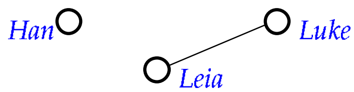
- More general and arbitrary than trees (trees are a restricted type of graph)

Directed/Undirected Graphs

- In *directed* graphs, edges have a specific direction:



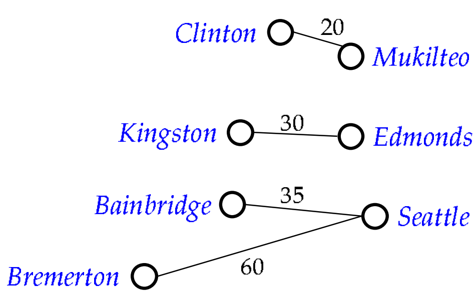
- In *undirected* graphs, they don't:



- Vertices u and v are *adjacent* if $(u,v) \in E$

Weighted Graphs

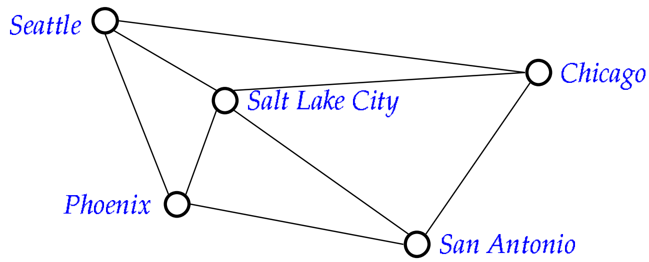
weighted graphs store a weight with each edge:



(nodes sometimes have weights too…)

Paths

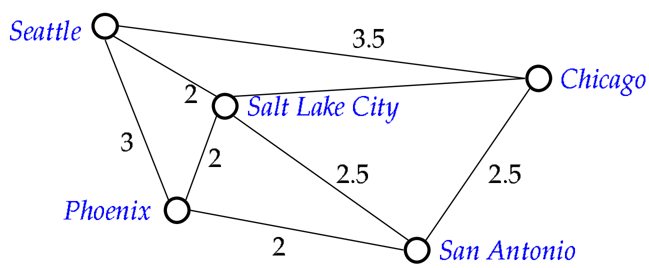
A *path* is a list of vertices $v_1, v_2, \ldots v_n$ such that $(v_i, v_{i+1}) \in E$.



p = *Seattle, Salt Lake City, Phoenix, San Antonio, Phoenix, Seattle*

Path Length and Cost

- The *length* of a path is the number of edges
- Its *cost* is the sum of the edges' weights



length(p) = 5 cost(p) = 11

Simple Paths and Cycles

A *simple path* repeats no vertices (except the first can also be the last):

– *Seattle, Salt Lake City, Phoenix, San Antonio*

– *Seattle, Salt Lake City, San Antonio, Phoenix, Seattle*

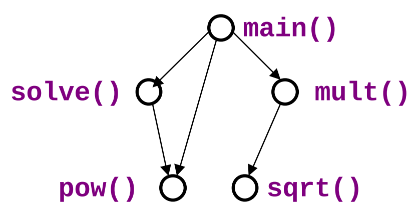
A *cycle* is a path that starts and ends at the same node

– *Seattle, Salt Lake City, San Antonio, Phoenix, Seattle*

(For undirected graphs edges cannot appear twice)

Directed Acyclic Graphs

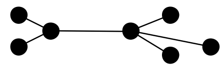
Directed Acyclic Graphs (*DAG*s) are directed graphs with no cycles



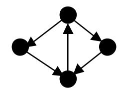
trees $\subset$ *DAGs* $\subset$ *directed graphs*

Connectivity

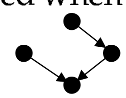
- Undirected graphs are *connected* if there is a path between any two vertices



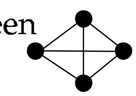
- Directed graphs are *strongly connected* if there is a path between any two vertices



- It is *weakly connected* if it's connected when direction is ignored



- A *complete* graph is one that has an edge between every pair of vertices



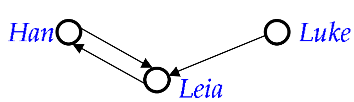
Graph Implementations

Adjacency Matrix: A $|V| \times |V|$ array in which:

– element (**u**,**v**) is 1 if there is an edge (u,v)

– it is 0 otherwise

– for weighted graphs, store weights rather than 1/0

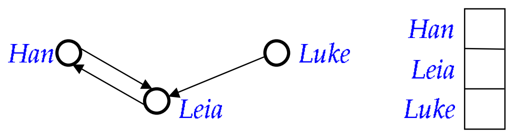


	Han	*Leia*	*Luke*
Han			
Leia			
Luke			

– space requirements?

Graph Implementations

Adjacency Lists: A $|V|$-ary array in which each entry stores a list of all adjacent vertices



Space requirements?

How could we index into an adjacency list or matrix when nodes are named?

Graph Applications

- Storing things that are graphs by nature (AI)
  - distances between cities
  - airline flights, travel options
  - relationships between people, things
  - distances between rooms in the game Clue
- Compilers
  - *callgraph* – which functions call which other ones
  - *dependence graphs* – which variables are defined and used at which statements