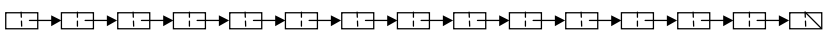
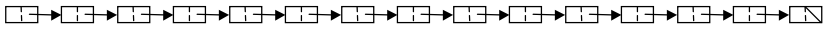


CSE 373: Asymptotic Analysis

Chapter 2



Recall: `FindMajor()`

```
/* print students in a class with major */
void FindMajor(class c,dept major) {
  int i;

  for (i=0;i<num_students;i++) {
    if (c[i].major == major) {
      printf("%s %s\n",c[i].first,c[i].last);
    }
  }
}
```

How fast is this routine?

Exact Times are Tricky

How much time does `FindMajor()` require?

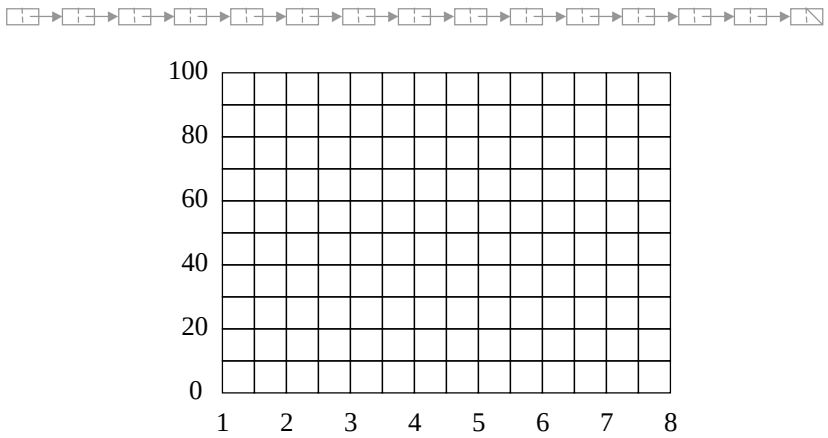
- number of iterations × work per iteration:

Simplifying Assumption

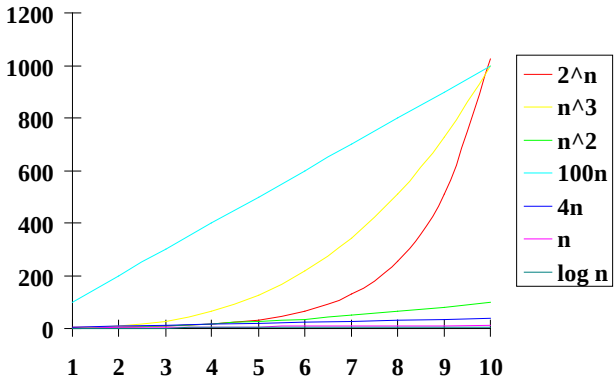
Constants are insignificant compared to the *asymptotic* behavior of the program

- expressed as a function of the problem size
- expressed using functions like: n, n^2, $\log n$, 2^n, etc.

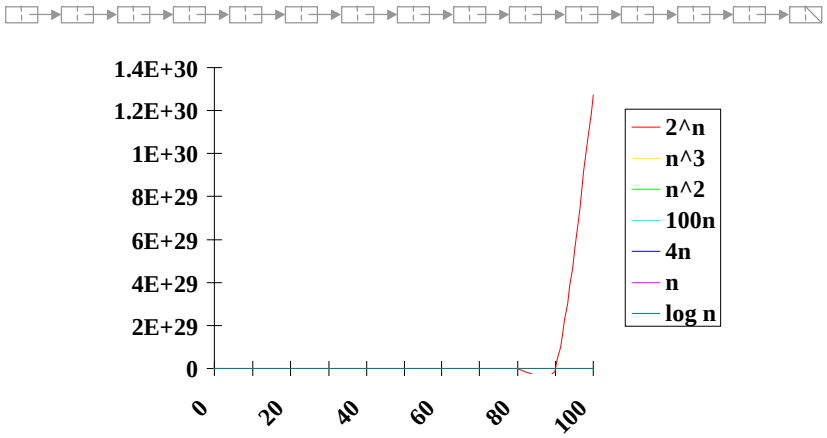
Getting some Intuition...



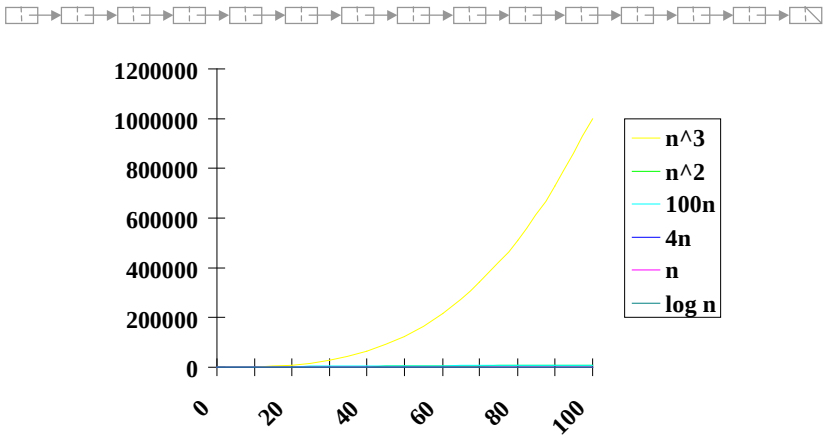
Using the Computer...



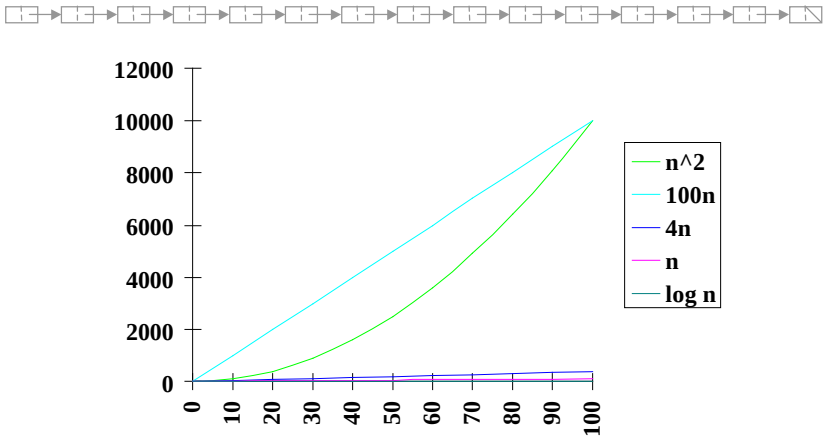
On A Larger Scale...



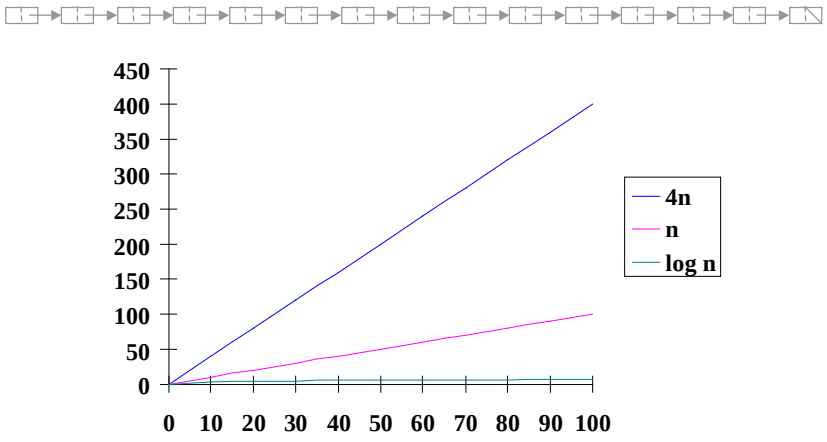
Ignoring 2^n



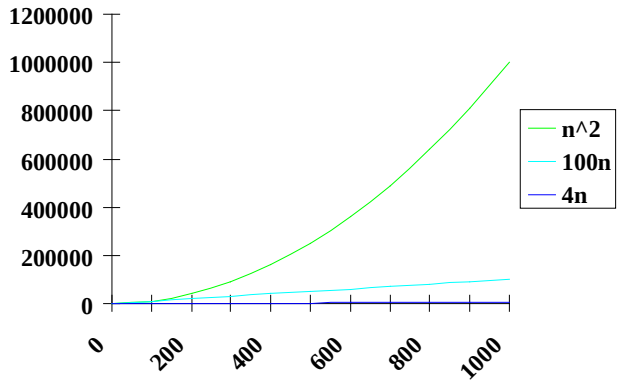
Ignoring n^3



Smallest Functions Only



On Yet a Larger Scale



The Moral

Performance can be broken down into *primary* and *secondary effects*

- *primary effects*: asymptotic growth pattern
- *secondary effects*: constant factors, less significant terms

- In this class, we'll mainly be concerned with primary effects (*asymptotic analysis*)
- In the real world, secondary effects are also often worth paying attention to (*after* the primary ones)

Formally…

Given an algorithm whose running time is T(n)…

- T(n) = O($f(n)$) if there are positive constants c and n_0 such that T(n) ≤ $c \cdot f(n)$ for all $n \geq n_0$
  - log n, n, $100n$ = O(n)
- T(n) = Ω($f(n)$) if there are positive constants c and n_0 such that T(n) ≥ $c \cdot f(n)$ for all $n \geq n_0$
  - n, n^2, $100 \cdot 2^n$, $n^3 \log n$ = Ω(n)
- T(n) = Θ($f(n)$) if T(n) = O($f(n)$) and T(n) = Ω($f(n)$)
  - n, $2n$, $100n$, $n + \log n$ = Θ(n)

Typical Growth Rates (in order)

constant:	O(1)
logarithmic:	O(log n)
log-squared:	O(log^2 n)
linear:	O(n)
"n log n":	O($n \cdot \log n$)
quadratic:	O(n^2)
cubic;	O(n^3)
exponential:	O(2^n)

General Rules of Thumb

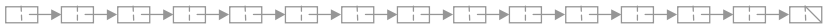
- Constant factors can always be dropped
  - $5n = O(n)$
- In sums, smaller terms can always be dropped
  - $3n \cdot \log n + n^2 + \log n = O(n^2)$

Analyzing Code

- *C/C++ ops* – constant value
- *consecutive statements* – add individual costs
- *loops* – multiply cost of loop body by number of iterations
- *conditionals* – maximum cost of branches
- *function calls* – evaluate cost of function body

Above all, use your brain

Reconsider: `FindMajor()`



```
/* print students in a class with major */
void FindMajor(class c,dept major) {
  int i;
  for (i=0;i<num_students;i++) {
    if (c[i].major == major) {
      cout << c[i].first << c[i].last;
    }
  }
}
```

How fast is this routine? At best?

At worst?

On average?

`FindAMajor()`

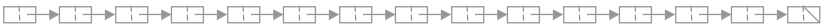
```
/* return pointer to a student in major */
student *FindAMajor(class c,dept major) {
  int i;
  for (i=0;i<num_students;i++) {
    if (c[i].major == major) {
      return &(c[i]);
    }
  }
}
```

What's the best case for this routine?

The worst case?

The average case?

Asymptotic Analysis



- Determine what characterizes a problem's size
- Express how much time and memory an algorithm requires as a function of its problem size using O(), Ω(), or Θ()
  - worst case
  - best case
  - average case
  - common case
  - overall

Examples from Lecture

	Prob Size	*Space*	*Best Time*	*Worst Time*

UW Registry
`TakingClass()`
`PrintSchedule()`
`MakeClassList()`				

iterative `fact()`				
recursive `fact()`				
`PaintFill()`				
linear search				
binary search				