CSE/EE 373 Autumn 1994
Data Structures and Algorithm
Assignment # 1 Common Mistake Solutions

1. Estimate

$$\sum_{i=\lfloor \frac{n}{2} \rfloor}^{n} \frac{1}{i}$$

Answer:

$$H_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}$$

H_n is called the nth **harmonic number**. Although the values of the H_n increase more slowly As n increases, the series does not converge. H_n grows with n in a logrithmic fashion; to be precise $H_n \approx \ln n + \gamma, where \gamma = 0.577\ldots$ is a number called **Euler's Constant**.

Case 1, n = even number

$$\lfloor \frac{n}{2} \rfloor = \frac{n}{2}$$

$$\sum_{i=\lfloor \frac{n}{2} \rfloor}^{n} \frac{1}{i} = \ln n + \gamma - (\ln(\frac{n}{2} - 1) + \gamma) = \ln n - \ln(\frac{n}{2} - 1) = \ln(\frac{2n}{n-2})$$

Case 2, n = odd number

$$\lfloor \frac{n}{2} \rfloor = \frac{n-1}{2}$$

$$\sum_{i=\lfloor \frac{n}{2} \rfloor}^{n} \frac{1}{i} = \ln n + \gamma - (\ln(\frac{n-1}{2} - 1) + \gamma) = \ln n - \ln(\frac{n-1}{2} - 1) = \ln(\frac{2n}{n-3})$$