Section 05: Heaps + Exam 1 Review

Heap Problems

1. Ternary Heaps

Consider the following sequence of numbers:

5, 20, 10, 6, 7, 3, 1, 2

(a) Insert these numbers into a min-heap where each node has up to *three* children, instead of two.

(So, instead of inserting into a binary heap, we're inserting into a ternary heap.)

Draw out the tree representation of your completed ternary heap.

(b) Draw out the array representation of the above tree. In your array representation, you should start at index 0 (not index 1).

(c) Given a node at index i, write a formula to find the index of the parent.

(d) Given a node at index i, write a formula to find the j-th child. Assume that $0 \leq j < 3$.

2. Heaps – More Basics

(a) Insert the following sequence of numbers into a *min heap*:

[10, 7, 15, 17, 12, 20, 6, 32]

(b) Now, insert the same values into a *max heap*.

(c) Now, insert the same values into a *min heap*, but use Floyd's `buildHeap` algorithm.

(d) Insert 1, 0, 1, 1, 0 into a *min heap*.

(e) Call `removeMin` on the min heap stored as the following array: $[2, 5, 7, 8, 10, 9]$

3. Sorting and Reversing (with Heaps)

(a) Suppose you have an array representation of a heap. Must the array be sorted?

(b) Suppose you have a sorted array (in increasing order). Must it be the array representation of a valid min-heap?

(c) You have an array representation of a min-heap. If you reverse the array, does it become an array representation of a max-heap?

(d) Describe the most efficient algorithm you can think of to convert the array representation of a min-heap into a max-heap. What is its running time?

4. Project Prep: Contains

You just finished implementing your heap of `int`s when your boss tells you to add a new method called `contains`. Your solution should not, in general, examine every element in the heap(do it recursively!)

```
public class DankHeap {
    // NOTE: Data starts at index 0!
    private int[] heapArray;
    private int heapSize;

    // Other heap methods here....

    /**
     * examine whether element k exists in the heap
     * @param int k, the element to find.
     * @return true if found, false otherwise
     */
    public boolean contains(int k) {
        // TODO!
    }
}
```

(a) How efficient do you think you can make this method?

(b) Write code for `contains`. Remember that `heapArray` starts at index 0!

5. Challenge: Debugging Heaps of Problems

For this problem, we will consider a hypothetical hash table that uses linear probing and implements the `IDictionary` interface. Specifically, we will focus on analyzing and testing one potential implementation of the `remove` method.

(a) Come up with at least 4 different test cases to test this `remove(...)` method. For each test case, describe what the expected outcome is (assuming the method is implemented correctly).

Try and construct test cases that check if the `remove(...)` method is correctly using the key's hash code. (You may assume that you can construct custom key objects that let you customize the behavior of the `equals(...)` and `hashCode()` method.)

(b) Now, consider the following (buggy) implementation of the `remove(...)` method. List all the bugs you can find.

```
public class LinearProbingDictionary<K, V> implements IDictionary<K, V> {
    // Field invariants:
    //
    // 1. Empty, unused slots are null
    // 2. Slots that are actually being used contain an instance of a Pair object

    private Pair<K, V>[] array;

    // ...snip...

    public V remove(K key) {
        int index = key.hashCode();

        while ((this.array[index] != null) && !this.array[index].key.equals(key)) {
            index = (index + 1) % this.array.length;
        }

        if (this.array[index] == null) {
            throw new NoSuchKeyException();
        }
        V returnValue = this.array[index].value;
        this.array[index] = null;
        return returnValue;
    }
}
```

(c) Briefly describe how you would fix these bug(s).

Exam 1 Review

6. Selecting ADTs and data structures

For each of the following scenarios, choose:

(a) An ADT: `Stack` or `Queue`

(b) A data structure: `array list` or `linked list with front` or `linked list with front and back`

Justify your choice.

(a) You're designing a tool that checks code to verify all opening brackets, braces, parenthesis, etc... have closing counterparts.

(b) Disneyland has hired you to find a way to improve the processing efficiency of their long lines at attractions. There is no way to forecast how long the lines will be.

(c) A sandwich shop wants to serve customers in the order that they arrived, but also wants to look ahead to know what people have ordered and how many times to maximize efficiency in the kitchen.

7. Best case and worst case runtimes

For the following code snippet give the big-Θ bound on the worst case runtime as well the big-Θ bound on the best case runtime, in terms of n the size of the input array.

```
void print(int[] input) {
    int i = 0;
    while (i < input.length - 1) {
        if (input[i] > input[i + 1]) {
            for (int j = 0; j < input.length; j++) {
                System.out.println("uh I don't think this is sorted plz help");
            }
        } else {
            System.out.println("input[i] <= input[i + 1] is true");
        }
        i++;
    }
}
```

8. Big-O, Big-Omega True/False Statements

For each of the statements determine if the statement is true or false. You do not need to justify your answer.

(a) $n^3 + 30n^2 + 300n$ is $\mathcal{O}(n^3)$

(b) $nlog(n)$ is $\mathcal{O}(log(n))$

(c) $n^3 - 3n + 3n^2$ is $\mathcal{O}(n^2)$

(d) 1 is $\Omega(n)$

(e) $.5n^3$ is $\Omega(n^3)$

9. Eyeballing Big-Θ bounds

For each of the following code blocks, what is the worst-case runtime? Give a big-Θ bound. You do not need to justify your answer.

(a)
```
void f1(int n) {
    int i = 1;
    int j;
    while(i < n*n*n*n) {
        j = n;
        while (j > 1) {
            j -= 1;
        }
        i += n;
    }
}
```

(b)
```
int f2(int n) {
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
```

```
            System.out.println("j = " + j);
        }
        for (int k = 0; k < i; k++) {
            System.out.println("k = " + k);
            for (int m = 0; m < 100000; m++) {
                System.out.println("m = " + m);
            }
        }
    }
}
```

(c)
```
int f3(n) {
    count = 0;
    if (n < 1000) {
        for (int i = 0; i < n; i++) {
            for (int j = 0; j < n; j++) {
                for (int k = 0; k < i; k++) {
                    count++;
                }
            }
        }
    } else {
        for (int i = 0; i < n; i++) {
            count++;
        }
    }
    return count;
}
```

(d)
```
void f4(int n) {
    // NOTE: This is your data structure from the first project.
    LinkedDeque<Integer> deque = new LinkedDeque<>();
    for (int i = 0; i < n; i++) {
        if (deque.size() > 20) {
            deque.removeFirst();
        }
        deque.addLast(i);
    }
    for (int i = 0; i < deque.size(); i++) {
        System.out.println(deque.get(i));
    }
}
```

10. Challenge: Design

Imagine a database containing information about all trains leaving the Washington Union station on Monday. Each train is assigned a departure time, a destination, and a unique 8-digit train ID number.

What data structures you would use to solve each of the following scenarios. Depending on scenario, you may need to either (a) use multiple data structures or (b) modify the implementation of some data structure.

Justify your choice.

(a) Suppose the schedule contains 200 trains with 52 destinations. You want to easily list out the trains by destination.

(b) In the question above, trains were listed by destination. Now, trains with the same destination should further be sorted by departure time.

(c) A train station wants to create a digital kiosk. The kiosk should be able to efficiently and frequently complete look-ups by train ID number so visitors can purchase tickets or track the location of a train. The kiosk should also be able to list out all the train IDs in ascending order, for visitors who do not know their train ID.

Note that the database of trains is not updated often, so the removal and additions of new trains happen infrequently (aside from when first populating your chosen DS with trains).